Example: Sample ***Means***

1. Suppose you have 4 students whose ages are 18, 19, 20, and 25. The population are the values of their ages $\{18, 19, 20, 25\}$.

(a). Find the mean of their ages (i.e. the mean, μ, of the population.)

(b). The first column in the table below left consists of all possible samples of size 2 *with replacement*. Find the mean, $\bar{x}$, of each sample and enter it in the second column.

[Table for part (b)]

Sample	Sample Mean $\bar{x}$
19, 18	18.5
19, 19	19.0
19, 20	19.5
19, 25	22.0
20, 18	19.0
20, 19	19.5
20, 20	20.0
20, 25	22.5
25, 18	21.5
25, 19	22.0
25, 20	22.5
25, 25	25.0

[Table for part (c)]

Sample Mean $\bar{x}$	Probability
18.0	
18.5	
19.0	
19.5	2/16
20.0	1/16
21.5	2/16
22.0	2/16
22.5	2/16
25.0	1/16

(c). Using the table of the Sample Means (above left), complete the probability distribution table (above right).

(d). Note that the table from part (c) describes a probability distribution for the sample mean $\bar{x}$. Find the mean of this probability distribution (i.e. Find $\sum \bar{x} \cdot P(\bar{x})$ and call in $\mu_{\bar{x}}$)

(e). How do μ from part (a) and $\mu_{\bar{x}}$ from part (d) compare?

2. Using the same population $\{18, 19, 20, 25\}$, the data below gives all possible samples of size 3 *with replacement.*

Sample	18, 18, 18	18, 18, 19	18, 18, 20	18, 18, 25	18, 19, 18	18, 19, 19	18, 19, 20	18, 19, 25
Mean $\bar{x}$	**18.0**	**18.3**	**18.7**	**20.3**	**18.3**	**18.7**	**19.0**	**20.7**
Sample	18, 20, 18	18, 20, 19	18, 20, 20	18, 20, 25	18, 25, 18	18, 25, 19	18, 25, 20	18, 25, 25
Mean $\bar{x}$	**18.7**	**19.0**	**19.3**	**21.0**	**20.3**	**20.7**	**21.0**	**22.7**
Sample	19, 18, 18	19, 18, 19	19, 18, 20	19, 18, 25	19, 19, 18	19, 19, 19	19, 19, 20	19, 19, 25
Mean $\bar{x}$	**18.3**	**18.7**	**19.0**	**20.7**	**18.7**	**19.0**	**19.3**	**21.0**
Sample	19, 20, 18	19, 20, 19	19, 20, 20	19, 20, 25	19, 25, 18	19, 25, 19	19, 25, 20	19, 25, 25
Mean $\bar{x}$	**19.0**	**19.3**	**19.7**	**21.3**	**20.7**	**21.0**	**21.3**	**23.0**
Sample	20, 18, 18	20, 18, 19	20, 18, 20	20, 18, 25	20, 19, 18	20, 19, 19	20, 19, 20	20, 19, 25
Mean $\bar{x}$	**18.7**	**19.0**	**19.3**	**21.0**	**19.0**	**19.3**	**19.7**	**21.3**
Sample	20, 20, 18	20, 20, 19	20, 20, 20	20, 20, 25	20, 25, 18	20, 25, 19	20, 25, 20	20, 25, 25
Mean $\bar{x}$	**19.3**	**19.7**	**20.0**	**21.7**	**21.0**	**21.3**	**21.7**	**23.3**
Sample	25, 18, 18	25, 18, 19	25, 18, 20	25, 18, 25	25, 19, 18	25, 19, 19	25, 19, 20	25, 19, 25
Mean $\bar{x}$	**20.3**	**20.7**	**21.0**	**22.7**	**20.7**	**21.0**	**21.3**	**23.0**
Sample	25, 20, 18	25, 20, 19	25, 20, 20	25, 20, 25	25, 25, 18	25, 25, 19	25, 25, 20	25, 25, 25
Mean $\bar{x}$	**21.0**	**21.3**	**21.7**	**23.3**	**22.7**	**23.0**	**23.3**	**25.0**

(a). Complete the following probability distribution table.

$\bar{x}$	Probability
18.0	
18.3	
18.7	
19.0	7/64
19.3	6/64
19.7	3/64
20.0	1/64
20.3	3/64
20.7	6/64
21.0	9/64
21.3	6/64
21.7	3/64
22.7	3/64
23.0	3/64
23.3	3/64
25.0	1/64

(b). Find the mean of this probability distribution (i.e. Find $\sum \bar{x} \cdot P(\bar{x})$ and call in $\mu_{\bar{x}}$)

(c). How do μ from #1 part (a) and $\mu_{\bar{x}}$ from #2 part (b) compare?

Example: Sample ***Ranges***

3. Suppose you have 4 students whose ages are 18, 19, 20, and 25. The population are the values of their ages $\{18, 19, 20, 25\}$.

(a). Find the range of their ages (i.e. the range of the population.)

(b). The first column in the table below left consists of all possible samples of size 2 *with replacement*. Find the range of each sample and enter it in the second column.

[Table for part (b)]

Sample	Sample Range
18, 18	
18, 19	
18, 20	
18, 25	
19, 18	1
19, 19	0
19, 20	1
19, 25	6
20, 18	2
20, 19	1
20, 20	0
20, 25	5
25, 18	7
25, 19	6
25, 20	5
25, 25	0

[Table for part (c)]

Sample Range	Probability
0	
1	
2	
5	2/16
6	2/16
7	2/16

(c). Using the table of the Sample Ranges (above left), complete the probability distribution table (above right).

(d). Note that the table from part (c) describes a probability distribution for the Sample Ranges. Find the mean of this probability distribution (i.e. Find $\sum x \cdot P(x)$ and call it the mean of the Sample Ranges.

(e). How does the population range from part (a) and the mean of the Sample Ranges from part (d) compare?