

1. So the solution to the transient problem is

$$w(x, t) = \tilde{a}_0 + \sum_{n=1}^{\infty} a_n \cos(\lambda_n x) e^{-\lambda_n^2 kt} \quad \text{where } \lambda_n = \frac{n\pi}{L}$$

(a). Use the initial condition $w(x, 0) = g(x) = f(x) - v(x) = f(x) - C_2$ to find \tilde{a}_0 .

Cosine Series \implies

$$\tilde{a}_0 = \frac{1}{L} \int_0^L g(x) dx = \frac{1}{L} \int_0^L dx$$

$$= \frac{1}{L} \int_0^L f(x) dx - \frac{1}{L} \int_0^L C_2 dx$$

$$= \frac{1}{L} \int_0^L f(x) dx - \left. \frac{1}{L} \int_0^L f(x) dx - \frac{1}{L} C_2(L - 0) \right|_0^L$$

$$= \frac{1}{L} \int_0^L f(x) dx -$$

$$= a_0 - C_2$$

where a_0 is the 0th coefficient for _____, not _____.

(b). Use the result of part(a) to write the solution for the transient $w(x, t)$.

(c). Write the full solution for $u(x, t)$ (simplify).

$$u(x, t) = v(x) + w(x, t) =$$

where $a_0 =$ _____ and $a_n =$ _____

(d). Compare the form of the solution for the transient $w(x, t)$ and the full solution $u(x, t)$.

Helpful time-saving tip

If the PDE and BC's are already homogeneous,