Ex: Steady state heat equation in a thin rod [done in class].

Ex: Deflection of a string/cable (e.g. )

- Let $u(x)$ be the

- Let $u(0) =$ and $u(L) =$

Assumptions:

- $f(x)$ is the ______________________________________ .
- The cable hangs in ________________ $\Longrightarrow$ __________________ .

Let $T(x) =$

Let $\phi(x) =$
with respect to the $x$-axis

For convenience, let $T_1 =$ __________ and $T_2 =$ _______________ and $\alpha =$ __________ and $\beta =$ _______________

Newton's 2nd Law:

Horizontal:

Vertical:

From (1): $T_1 \cos\alpha = T_2 \cos\beta \quad \Rightarrow$

$T_1 \cos\alpha =$ ________ and $T_2 \cos\beta =$ ________ $\Rightarrow$ $T_1 =$ and $T_2 =$

Substitute into (2): $\quad - \qquad \sin\alpha + \qquad \sin\beta = f(x)\Delta x$

$\Rightarrow$

Substitute $\phi(x)$ and $\phi(x + \Delta x)$ back in:

$T\left[\tan \qquad - \tan \qquad \right] = f(x)\Delta x$

Replace the tangents with ________________ : $\quad T\left[ \qquad - \qquad \right] = f(x)\Delta x$

Rearrange: $\quad T \cdot \dfrac{\qquad - \qquad}{\Delta x} = f(x)$

Take the limit: $\quad T \cdot \lim_{\Delta x \to 0} \dfrac{\qquad - \qquad}{\Delta x} = f(x)$

Recognize: $\quad T \cdot \qquad = f(x)$

BVP:

Possibilities for the load $f(x)$

Ex: Load is distributed uniformly in the $x$-direction. $\Rightarrow$

$$\frac{d^2u}{dx^2} = \frac{w}{T} \qquad \text{on } 0 < x < L \qquad u(0) = h_0 \quad u(L) = h_1$$

Solve:

Ex: Cable hangs under its own weight $w$ in units of weight per unit of length of cable.

BVP: $T \cdot \dfrac{d^2u}{dx^2} = w \cdot$ on $0 < x < L$ $\quad u(0) = h_0 \quad u(L) = h_1$ $\qquad$ ( $\qquad$ )

Solution given by $\qquad$ ( $\qquad$ )

Ex: Buckling of a Column

$$\frac{d^2u}{dx^2} + \lambda^2 u = 0 \qquad u(0) = 0 \qquad u(L) = 0$$

Solve:

Note: This last example is an ____________________ :

- ________________ Differential Equation with a parameter $\lambda$

  (The values of $\lambda$ for which there is a nontrivial solution are called the ______________ .)

- ________________ Boundary Conditions