**1.** Classify each of the following differential equations as ***(i)*** an ODE or PDE; ***(ii)*** linear or nonlinear; and ***(iii)*** homogeneous or nonhomogeneous. Also, clearly state the ***(iv)*** order and the ***(v)*** independent and dependent variables.

**(a).** $3\dfrac{d^2x}{dt^2} + 4\dfrac{dx}{dt} + 9x = 2\cos 3t$

**(b).** $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$

**(c).** $y\left[1 + \left(\dfrac{dy}{dx}\right)^2\right] = C$

**(d).** $\dfrac{\partial N}{\partial t} = \dfrac{\partial^2 N}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial N}{\partial r} + kN$

**2.** Solve the following first-order equations using the indicated method.

**(a).** $\dfrac{du}{dt} = u^2 \sin t$ [Separation of Variables]

**(b).** $\dfrac{du}{dt} = ku$ for constant $k$. [Separation of Variables]

**(c).** $\dfrac{du}{dt} = k(t)u$ for arbitrary function $k(t)$. (answer will contain an integral) [Separation of Variables]

**(d).** $\dfrac{du}{dt} + \dfrac{1}{t}u = 4t^2$ [Integrating Factor $\mu(t) = e^{\int p(t)\, dt}$]

**3.** Given the general second-order linear homogeneous equation with constant coefficients $\dfrac{d^2u}{dt^2} + k\dfrac{du}{dt} + pu = 0$

**(a)**. Substitute the possible solution $u(t) = e^{mt}$ into the equation and simplify.

What is this new algebraic equation called?

**(b)**. Solve this equation for $m$ (i.e. find the roots $m_1, m_2$).

**(c)**. For each case below, write down the general solution $u(t)$:

| | Roots of the Characteristic Equation | General Solution to the Differential Equation |
|---|---|---|
| Case 1 | Real, distinct roots: $m_1 \neq m_2$ | |
| Case 2 | Real, repeated roots: $m_1 = m_2$ | |
| Case 3 | Complex conjugate roots: $m_1 = \alpha + i\beta, m_2 = \alpha - i\beta$ | |
| (subcase) | Purely imaginary when $\alpha = 0$: $m_1 = i\beta, m_2 = -i\beta$ | |

**4.** Find the general solution for the following differential equations. [Recall, that for higher-order equations with more than 2 roots, each root contributes the solution type above (multiply by an extra $t$ each time it is repeated).]

**(a)**. $u'' + 2u' + u = 0$

**(b)**. $\dfrac{d^2u}{dt^2} - \dfrac{du}{dt} + 7u = 0$

**(c)**. $\dfrac{d^4u}{dt^4} + 5\dfrac{d^2u}{dt^2} - 36u = 0$

**(d)**. $D^3(D-5)^2(D^2+1)[u] = 0$ $\left[D \equiv \dfrac{d}{dx}\right]$