



**6. (a).** If the characteristic equation for a second order equation has the two real roots  $\lambda$  and  $-\lambda$ , why can we write the solution using exponentials  $\{e^{\lambda t}, e^{-\lambda t}\}$  or using the hyperbolic functions  $\{\sinh \lambda t, \cosh \lambda t\}$ ?

**(b).** Note that the roots for  $u'' - 4u = 0$  are 2, -2. Given the initial conditions  $u(0) = 5$  and  $u'(0) = 4$ ,

*(i)* Find the solution using the general solution in exponential form  $u(t) = C_1 e^{2t} + C_2 e^{-2t}$

*(ii)* Find the solution using the general solution in hyperbolic form  $u(t) = C_1 \cosh 2t + C_2 \sinh 2t$

Graph both solutions on your calculator to verify that they are, indeed, the same solution.

*(iii)* From parts *(i)* and *(ii)*, explain why it could be advantageous to write the general solution in hyperbolic form.