This portion of the test is take-home and you are on your honor to work alone. By turning in the assignment with your name, you are verifying that you have worked alone on this problem.

1. (26 pts) Given the problem

$$\text{PDE:} \quad \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - 16u, \qquad\qquad 0 < x < 1, \ t > 0$$

$$\text{BC:} \quad u(0,t) = 1, \qquad \frac{\partial u}{\partial x}(1,t) = 0, \qquad t > 0$$

$$\text{IC:} \quad u(x,0) = 1 - x, \qquad\qquad 0 < x < 1$$

(a). Determine the replacement equations for arbitrary n. Clearly indicate for which values of i each equation holds. [Use central difference replacement for derivatives in the boundary conditions.]

(b). Find the stability bound on r. [Hint: If Δt appears in your equation(s), replace it using $\Delta t = r \cdot (\Delta x)^2$.]

(c). Use Maple to solve the equation numerically for $n = 16$. Use 200 time steps. Turn in the Maple solution via email. [Note: Add 0.012 to your r value and re-run the problem. You should see numerical instabilities appear.]