Name: ______________________________ **Exam 2**

Math 342 Applied Analysis – Crawford **20 April 2016**

Score	
1	/32
2	/26
3	/20
4	/26
Total	/100

- Books and notes are not allowed. You may use calculators, integral tables, and coefficient formulas.
- Clearly indicate your answers.
- ***Show all your work*** – partial credit may be given for written work.
- Good Luck!

1. (32 pts). Given the problem

$$\text{PDE:} \quad \frac{1}{k}\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - r, \qquad 0 < x < L,\ t > 0$$

$$\text{BC:} \quad \frac{\partial u}{\partial x}(0,t) = 0, \qquad u(L,t) = T, \qquad t > 0$$

$$\text{IC:} \quad u(x,0) = f(x), \qquad 0 < x < L$$

where r and T are constants,

(a). ***State and solve*** the full steady-state problem.

(b). Showing all your steps, clearly determine and state the full transient problem. [***Do not solve.*** (yet)]

(c). Use the method of Separation of Variables to solve the transient problem in part (b). [i.e. Determine and solve eigenvalue problem, etc.]

(d). Write the answer to the ***full problem*** given above in the form $u(x,t) = \ldots$. State any coefficient formulas.

2. (26 pts). Given the regular Sturm-Liouville problem

$$(x\phi')' + \lambda^2 \left(\frac{1}{x}\right) \phi = 0\,, \qquad 1 < x < b$$
$$\phi(1) = 0\,, \qquad \phi(b) = 0$$

(a). Show that $\phi(x) = c_1 \cos(\lambda \ln x) + c_2 \sin(\lambda \ln x)$ is the *general* solution to the differential equation.

(b). Find the eigenvalues λ_n and eigenfunctions ϕ_n.

(c). Write down the form of the eigenfunction expansion for $f(x) = 1$ using the eigenfunctions from part (b). Give explicit integral formula(s) for the coefficient(s) used in the expansion for $f(x) = 1$.
[***But do <u>not</u> evaluate the integrals!***]

3. (20 pts). Given $f(x) = \begin{cases} e^{-\alpha x}, & 0 < x < \infty \\ 0, & \text{otherwise} \end{cases}$ where α is a positive constant.

Find the Fourier Integral Representation for $f(x)$.
[Evaluate any integral(s) for the coefficients used in the integral representation.]

4. (26 pts). Take-Home Problem – separate sheet