1. Use integration to find a solution $y(x)$ to the following differential equation. Don't forget the integration constants.

$$\frac{d^4y}{dx^4} = 0$$

2. *Radioactive Decay*: The rate of decay of a substance is proportional to the amount of substance present. If $A(t)$ represents the amount present at time t, then the previous statement can be written as the following differential equation:

$$\begin{array}{cccc} \dfrac{dA}{dt} & = & k \cdot A, & \qquad\qquad k < 0 \\ \textit{rate} & \textit{is} & \textit{prop. to amount} & \end{array}$$

(a). Verify that the function $A(t) = e^{kt}$ is a solution to this differential equation by differentiating $A(t)$ and substituting into the equation. [i.e. Verify that $A(t) = e^{kt}$ makes the equation a true statement.]

(b). Similarly, verify that any function of the form $A(t) = Ce^{kt}$ is a solution where C is an arbitrary constant.

3. Given the function $\phi(x)$, show that $y = \phi(x)$ is a solution to the given differential equation by differentiating and substituting into the equation.

(a). $y' + \frac{1}{x}y = 0, \;\; \phi(x) = \frac{1}{x} = x^{-1}$

(b). $y'' + 2y' - 3y = 0, \;\; \phi(x) = C_1e^{-3x} + C_2e^{x}$ for arbitrary constants C_1, C_2

4. Using the in-class example and problems 1 and 2 on this worksheet, complete the following table and answer the questions below. [Note: k and g are NOT arbitrary constants. They are given in the problem statement.]

Differential Equation	Order	General Solution (w/arbitrary constants)	# of Arbitrary Constants
$\frac{dA}{dt} = k \cdot A$		$A(t) = Ce^{kt}$	
$\frac{d^2h}{dt^2} = -g$			
$\frac{d^4y}{dx^4} = 0$			

How many arbitrary constants do you expect to get for an nth-order differential equation?

5. From #3b, the **general solution** is $y(x) = C_1e^{-3x} + C_2e^x$ which has 2 unknown constants, C_1 and C_2.

(a). Differentiate $y(x) = C_1e^{-3x} + C_2e^x$ to obtain $y'(x)$.

(b). Given the **initial conditions** $y(0) = 1$ and $y'(0) = 5$ [i.e. When $x = 0$, $y = 1$ and $y' = 5$], substitute these values into the general solution $y(x)$ and its derivative $y'(x)$ to determine new *algebraic* equations that involve *only* the unknown constants C_1 and C_2.

How many equations do you get for these unknowns? Why is that helpful?

(c). Using the equations obtained in part (b), solve for the unknown constants C_1 and C_2

(d). Substitute these values for C_1 and C_2 into the general solution to obtain the specific solution for this set of initial conditions.

(e). How many initial conditions (pieces of info) do you expect to need so that you can solve for all of the the arbitrary constants in a general solution to an n^{th}-order differential equation?