

Mathematical Models describe _____ phenomena.

e.g., projectile motion, swinging pendulum, springs, seismic waves, population growth, radioactive decay, etc.

DEF A DIFFERENTIAL EQUATION is an _____ relating an _____ and _____ .

EX:

You have seen some DE Models in Calculus:

EX: Free falling object (gravity is the only force acting on it). Let $h(t)$ be the height of the object at time t .

Then _____ is the acceleration.

From Newton's 2nd Law:

- The solution to the DE is a _____
- _____ appeared in the general solution (due to integration).

Given the derivative of x with respect to t : $\frac{dx}{dt}$, we know that

- x is the _____ variable
- t is the _____ variable

EX: $\frac{d^2y}{dx^2} + ax\frac{dy}{dx} + by = 0$

EX: $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$

ORDINARY DIFFERENTIAL EQUATION (_____): The unknown function (dependent variable) depends

PARTIAL DIFFERENTIAL EQUATION (_____): The unknown function (dependent variable) depends

A SYSTEM OF DIFFERENTIAL EQUATIONS:

Ex: Populations: $x(t)$ is a prey population and $y(t)$ is a predator population.

The ORDER of a differential equation is _____ appearing in the equation.

EX: $\frac{dA}{dt} = -kA$

EX: $y'' - y \cdot y' = 0$

EX: $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$

A general n^{th} order ODE involves

- The independent variable (e.g. t)
- The dependent variable (e.g. y) _____

General form:

$$F(t, y, y', y'', \dots, y^{(n)}) = 0 \quad (1)$$

EX: $y^{(iv)} + 3 \sin(t)y'' + y = 0$

Alternate form:

$$y^{(n)} = f(t, y, y', y'', \dots, y^{(n-1)}) \quad (2)$$

EX:

Recall, linear equations in 2D & 3D:

$$ax + by + c = 0$$

$$ax + by + cz + d = 0$$

Examples of nonlinear equations:

A differential equation is LINEAR if

Otherwise, it is NONLINEAR.

EX: $3y'' + 2y' + y = 0$

EX: $\frac{d^2y}{dx^2} - y\frac{dy}{dx} = 0$

EX: $5x^2y'' + 3(\cos x)y = e^x$

DEF A function $\phi(x)$ is a solution to the ODE [form (1) or (2)] on an interval I given that

$$F(t, \phi, \phi', \phi'', \dots, \phi^{(n)}) = 0 \quad \text{or} \quad \phi^{(n)} = f(t, \phi, \phi', \phi'', \dots, \phi^{(n-1)}) \quad \text{on the interval } I.$$

EX: Verify that $\phi(x) = x^3 + x$ is a solution to $x^2y'' - 3xy' + 3y = 0$