Name: ______________________________________ **Exam 1**

Math 341 Differential Equations – Crawford **05 March 2020**

Score	
1	/8
2	/10
3	/10
4	/28
5	/16
6	/16
7	/14
Total	/100

Books and notes are not allowed. You may use a calculator and an integral table. ***Show all your work*** – partial credit may be given for written work.

Put all work and answers on the separately provided paper. Staple the exam on top.

Good Luck!

Calculator Number:

Formulas that may or may not be helpful

$$m\frac{dv}{dt} = mg - bv,\ v(0) = v_0 \quad \Longrightarrow \quad v(t) = \frac{mg}{b} + \left(v_0 - \frac{mg}{b}\right)e^{-\frac{b}{m}t} \quad \text{and } x(0) = x_0 \Longrightarrow x(t) = \frac{mg}{b}t + \frac{m}{b}\left(v_0 - \frac{mg}{b}\right)\left(1 - e^{-\frac{b}{m}t}\right) + x_0$$

$$\frac{dP}{dt} = -aP(P-K),\ \ P(0) = P_0 \Longrightarrow P(t) = \frac{P_0K}{P_0 + (K - P_0)e^{-aKt}}$$

1. (8 pts). Determine the longest $t-$interval on which a unique solution is guaranteed to exist. If a unique solution is not guaranteed to exist, clearly state so.
[Justify your answer, but do **<u>not</u>** attempt to solve the problem.]

$$(t-4)y' + (\ln t)y = 2t^2 + 1, \qquad y(1) = 8$$

2. (10 pts). Determine for which values of r, the function $\phi(t) = e^{rt}$ is a solution to $3y'' + 4y' - 4y = 0$.

3. (10 pts). The Gompertz model for population growth is given by the following equation.

$$\frac{dy}{dt} = 2y \ln\left(\frac{80}{y}\right)$$

(a). Sketch the phase line for this equation.

(b). Determine the equilibrium points and classify each one as stable or unstable.

(c). If the initial condition is $y(0) = 100$, what will happen to the solution as $t \to \infty$?

4. (28 pts). Solve the following differential equations and initial value problems.

(a). $\dfrac{dy}{dx} = \dfrac{1}{x - xy}$ IC: $y(1) = 3$. [Leave the final answer in implicit form.]

(b). $y' + \dfrac{1}{1+t}y = 2t$ $t > -1$ [Write the final answer in explicit form.]

5. (16 pts). Verify that the following differential equation is exact. Then find the solution.

$$\frac{dy}{dx} = -\frac{ax + by}{bx + cy} \qquad a, b, c \text{ are nonzero constants.}$$

6. (16 pts). Suppose a brine mixture of 2 kg salt per liter runs into a tank initially filled with 500 L of water containing 5 kg of salt. The brine enters the tank at a rate of 4 L/min. The solution is well-mixed in the tank and is pumped out at the same rate.

(a). Set up ***and*** solve the *initial value problem* for the amount $Q(t)$ of salt in the tank at time t.

(b). Find the amount of salt in the tank after 10 minutes.

(c). After 10 minutes, a leak develops in the tank and an *additional* 0.1 L/min of fluid flows out of the tank. Set up but do ***not*** solve the *initial value problem* for the amount $Q(t)$ of salt in the tank t minutes after the leaking begins.

7. (14 pts). When Luke decides he would rather fall into the abyss of a reactor shaft than join his father on the Dark Side, he luckily gets sucked into air vent pipes and eventually hangs on to a weather vane (with only one hand!) until he is rescued. But what if he hadn't been sucked into the air vent and continued to fall down the shaft. The gravitational force constant on Cloud City is 10 kg·m/s^2 the force due to air resistance in the shaft is $3.2|v|$. Luke has a mass of 70 kg and he falls from the top of the shaft which is 5 km (5000 m) tall.

(a). What is the terminal velocity of his fall?

(b). Find the equation that describes his position at time t.

(c). As $t \to \infty$, what function does his position approach?

(d). Use the function you found in part (c) to approximate the time he hits the bottom of the shaft.