

## Method of Variation of Parameters

Example:

$$a(x)y'' + b(x)y' + c(x)y = d(x)$$

$$y'' + y = \sec x$$

0. Put in standard form

$$y'' + p(x)y' + q(x)y = g(x). \quad \checkmark$$

1. Find fundamental solution set  $\{y_1, y_2\}$  for the homogeneous equation and compute the Wronskian.

$$y'' + y = 0$$

$$r^2 + 1 = 0 \Rightarrow r = \pm i \Rightarrow \{\cos x, \sin x\}$$

$$y'' + p(x)y' + q(x)y = 0.$$

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

2. Determine  $v_1(x)$  and  $v_2(x)$  by

(a) Simplifying the formulas

$$v_1' = \frac{-g(x)y_2(x)}{W[y_1, y_2](x)}$$

$$v_1' = \frac{-\sec x \sin x}{1} = -\tan x$$

$$v_2' = \frac{g(x)y_1(x)}{W[y_1, y_2](x)}$$

$$v_2' = \frac{\sec x \cos x}{1} = 1$$

(b) Then integrating to obtain  $v_1(x)$  and  $v_2(x)$ .

[For simplicity, take integration constants to be 0.]

$$v_1 = \int -\tan x \, dx = -\ln |\sec x| = \ln |\cos x|$$

$$v_2 = \int 1 \, dx = x$$

3. Substitute  $v_1(x)$  and  $v_2(x)$  into the form of

$$Y_p(x) = v_1(x)y_1(x) + v_2(x)y_2(x).$$

$$Y_p(x) = \ln |\cos x| \cdot \cos x + x \sin x$$

4. Form the general solution

$$y(x) = c_1 y_1(x) + c_2 y_2(x) + Y_p(x).$$

$$y(x) = c_1 \cos x + c_2 \sin x + \cos x \ln |\cos x| + x \sin x$$