Mathematical Models describe _______________ phenomena.

e.g., projectile motion, swinging pendulum, springs, seismic waves, population growth, radioactive decay, etc.

Def A Differential Equation is an ______________ relating an ______________________ and

________________________________ .

Ex:

You have seen some DE Models in Calculus:
Ex: Free falling object (gravity is the only force acting on it). Let $h(t)$ be the height of the object at time t.

Then _________________ is the acceleration.

From Newton's 2nd Law:

- The solution to the DE is a ______________________________
- ______________________ appeared in the general solution (due to integration).

Given the derivative of x with respect to t: $\dfrac{dx}{dt}$, we know that

- x is the ______________ variable
- t is the ________________ variable

Ex: $\dfrac{d^2y}{dx^2} + ax\dfrac{dy}{dx} + by = 0$

Ex: $\dfrac{\partial^2 u}{\partial t^2} - c^2\dfrac{\partial^2 u}{\partial x^2} = 0$

Ordinary Differential Equation (__________): The unknown function (dependent variable) depends

Partial Differential Equation (__________): The unknown function (dependent variable) depends

A System of Differential Equations:

Ex: Populations: $x(t)$ is a prey population and $y(t)$ is a predator population.

The <u>Order</u> of a differential equation is ____________________________ appearing in the equation.

<u>Ex</u>: $\frac{dA}{dt} = -kA$ <u>Ex</u>: $y'' - y \cdot y' = 0$ <u>Ex</u>: $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$

A general n^{th} order ODE involves

- The independent variable (e.g. t)
- The dependent variable (e.g. y) ________________________

General form:

$$F\left(t, y, y', y'', \ldots, y^{(n)}\right) = 0 \tag{1}$$

<u>Ex</u>: $y^{(iv)} + 3\sin(t)y'' + y = 0$

Alternate form:

$$y^{(n)} = f\left(t, y, y', y'', \ldots, y^{(n-1)}\right) \tag{2}$$

<u>Ex</u>:

Recall, linear equations in 2D & 3D: $ax + by + c = 0$ $ax + by + cz + d = 0$

Examples of nonlinear equations:

A differential equation is <u>LINEAR</u> if

Otherwise, it is <u>NONLINEAR</u>.

<u>EX</u>: $3y'' + 2y' + y = 0$

<u>EX</u>: $\dfrac{d^2y}{dx^2} - y\dfrac{dy}{dx} = 0$

<u>EX</u>: $5x^2y'' + 3(\cos x)y = e^x$

<u>DEF</u> A function $\phi(x)$ is a solution to the ODE [form (1) or (2)] on an interval I given that

$\phi, \phi', \phi'', \ldots, \phi^{(n)}$ and

$F\left(t, \phi, \phi', \phi'', \ldots, \phi^{(n)}\right) = 0$ or $\phi^{(n)} = f\left(t, \phi, \phi', \phi'', \ldots, \phi^{(n-1)}\right)$ on the interval I.

<u>EX</u>: Verify that $\phi(x) = x^3 + x$ is a solution to $x^2y'' - 3xy' + 3y = 0$