The Final Exam is comprehensive.

Use old exams and review sheets to study previous material not included on this review.

1. Use the integral definition to find the Laplace Transform of the given functions. Indicate any restrictions on s.

(a). $f(t) = t^2 e^{at}$, for positive constant a.

$F(s) = \dfrac{2}{(s-a)^3}$, $s > a$

(b). $f(t) = \begin{cases} 2t, & 0 \le t < 2 \\ 4, & 2 \le t < 6 \\ 2, & 6 \le t < 8 \\ 0, & \text{otherwise} \end{cases}$

$F(s) = -2\,\dfrac{e^{-8s}s + e^{-6s}s + e^{-2s} - 1}{s^2}$

2. Find the inverse Laplace Transform.

(a). $F(s) = \dfrac{3s}{s^2 - 16}$

$f(t) = 3\cosh(4t)$

(b). $F(s) = \dfrac{e^{-3s}}{s^2 - s - 6}$

$f(t) = \frac{1}{5} u_3(t) \left(-e^{-2(t-3)} + e^{3(t-3)}\right)$

3. For the given initial value problem, find the Laplace Transform $Y(s)$ of the solution $y(t)$.
[Only find $Y(s)$, do not solve for $y(t)$*.]

(a). $y'' + 4y' + 5y = 0, \qquad y(0) = 1, y'(0) = 0$

$Y(s) = \dfrac{s+4}{s^2 + 4s + 5}$

(b). $y'' - 3y' - 10y = g(t), \qquad y(0) = 0, y'(0) = 1$ where $g(t) = \begin{cases} 0, & 0 \le t < 3 \\ 2, & 3 \le t < 6 \\ 0, & t \ge 6 \end{cases}$

$Y(s) = \dfrac{s + 2e^{-3s} - 2e^{-6s}}{s(s^2 - 3s - 10)}$

*Note: The take-home problem could be like one of these problems, but actually solving the full problem. In which case, the solutions would be (on the answer version):

(a). $y(t) = e^{-2t}\cos t + 2e^{-2t}\sin t$ and

(b). $y(t) = -\frac{1}{7}e^{-2t} + \frac{1}{7}e^{5t} + u_3(t)\left[\frac{1}{7}e^{-2(t-3)} + \frac{2}{35}e^{5(t-3)} - \frac{1}{5}\right] - u_6(t)\left[\frac{1}{7}e^{-2(t-6)} + \frac{2}{35}e^{5(t-6)} - \frac{1}{5}\right]$

4. Solve the following differential equations and initial value problems using any technique from class.

(a). $y'' - 3y' - 4y = 3e^{2x} + 2\sin x + 4x^2$ $\quad y(x) = C_1 e^{4x} + C_2 e^{-x} - \frac{1}{2}e^{2x} + \frac{3}{17}\cos x - \frac{5}{17}\sin x - x^2 + \frac{3}{2}x - \frac{13}{8}$

(b). $y'' - 3y' - 4y = e^{-x}, \qquad y(0) = 1,\ y'(0) = 4$ $\quad y(x) = \frac{26}{25}e^{4x} - \frac{1}{25}e^{-x} - \frac{1}{5}xe^{-x}$

(c). $y' = y^2/x, \qquad y(1) = -2$ $\quad y(x) = \frac{1}{-\ln x - \frac{1}{2}}$

(d). $x^2y'' - xy' + 4y = 0$ $\quad y(x) = C_1 x\cos(\sqrt{3}\ln x) + C_2 x\sin(\sqrt{3}\ln x)$

(e). $(x+1)^2y'' - 5(x+1)y' + 9y = 0$ $\quad y = C_1(x+1)^3 + C_2(x+1)^3\ln(x+1)$

(f). $y' - \frac{1}{x}y = xe^x$ $\quad y(x) = xe^x + Cx$

(g). $(2x + 4xy)dx + (2x^2 - 2y)dy = 0$ $\quad x^2 + 2x^2y - y^2 + C = 0$

(h). $y'' + 2y' + 2y = 0, \qquad y(\pi/4) = 2,\ y'(\pi/4) = -2$ $\quad y(x) = e^{\pi/4}\sqrt{2}e^{-x}(\sin x + \cos x)$

(i). $4y'' + y = 2\sec\left(\frac{t}{2}\right)$ $\quad y = c_1\cos(t/2) + c_2\sin(t/2) + t\sin(t/2) + 2[\ln(\cos(t/2))]\cos(t/2)$

The Final Exam is comprehensive.

Use old exams and review sheets to study previous material not included on this review.