

Name: \_\_\_\_\_

Math 341 Differential Equations – Crawford

Exam 2  
17 April 2019

Score

|       |      |
|-------|------|
| 1     | /6   |
| 2     | /14  |
| 3     | /24  |
| 4     | /14  |
| 5     | /14  |
| 6     | /14  |
| 7     | /16  |
| Total | /100 |

- Books and notes (in any form) are not allowed.
- You may use calculators and the provided integral table.
- **Put all of your work and answers on other sheets of paper.** Include this sheet as a cover sheet.
- **Show all your work.** Partial credit may be given for written work.

Formulas that may or may not be helpful.

$$R = \sqrt{C_1^2 + C_2^2} \quad \tan \delta = \frac{C_2}{C_1}$$

$$\cos(A) + \cos(B) = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\cos(A) - \cos(B) = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

Good Luck!

1. (6 pts). Determine the longest interval in which the given initial value problem is guaranteed to have a unique solution. [Do not attempt to find the solution.]

$$(t-3)y'' + \ln|t|y' = t, \quad y(1) = 1, y'(1) = -4$$

2. (14 pts). Determine the **form only** of a particular solution  $y_p$ .

Do **NOT** evaluate the constants.

$$D(D-2)^2(D^2+1)[y] = t^2 + te^{2t} + e^t \sin t$$

3. (24 pts). Find the general solution for the following differential equations.

(a).  $y'' - 4y' + 7y = 0$

(b).  $x^2y'' - 5xy' + 9y = 0$

4. (14 pts). Solve the following initial value problem.

$$y'' + y' - 6y = 3t + 10e^{-3t} \quad y(0) = 0, y'(0) = 1$$

5. (14 pts). Given the initial value problem

$$y'' + 4y' + 4y = 0 \quad y(0) = 1, y'(0) = b$$

- (a). Find the solution in terms of  $b$ .
- (b). Determine the time  $t_M$  (in terms of  $b$ ) at which a maximum in the graph occurs.

6. (14 pts). Given that  $y_1(t) = \cos(3t)$  and  $y_2(t) = \sin(3t)$  are solutions to the corresponding linear homogeneous equation, **use variation of parameters** to find the particular solution for the following differential equation. Then write the general solution.

$$y'' + 9y = \sec(3t)$$

7. (16 pts). A mass of 12 g stretches a spring 4 cm. The mass is attached to a viscous damper with a damping constant of 360 dyn-s/cm. The mass is compressed 2 cm from equilibrium position and then released. [Note: The units are all consistent and  $g = 980 \text{ cm/s}^2$ .]

- (a). Set up, but do not solve, the initial value problem for this mass-spring system.
- (b). Solve for the general solution only, but do **not** solve for the constants using the initial conditions.
- (c). Determine the quasi-period.