1. Classify the following differential equation as ordinary (ODE) or partial (PDE). Determine the order and state whether it is linear or nonlinear. Also, indicate the independent and dependent variables.

$$\frac{d^3y}{dt^3} + \cos t \frac{dy}{dt} + y = t$$

3rd order ODE, linear, dependent variable: y, independent variable: x

2. Determine whether the given function (explicit solutions) or relation (implicit solutions) is a solution to the given differential equation.

(a). $y = 3\sin 2x + e^{-x}$, $y'' + 4y = 5e^{-x}$ — Differentiate and substitute: yes

(b). $x^2 - \sin(x+y) = 1$, $\dfrac{dy}{dx} = 2x\sec(x+y) - 1$ — Implicit Differentiation: yes

3. Determine whether the following initial value problems have a unique solution. If so, what is the largest interval or rectangle on which a unique solution exists.

(a). $\frac{dy}{dx} = 3x - \sqrt[3]{y-1}$, $y(1) = 2$ — $\partial f/\partial y$ discont. at $y = 1 \Rightarrow$ Unique solution guaranteed on $-\infty < x < \infty, 1 < y < \infty$

(b). $\frac{dy}{dx} = 3x - \sqrt[3]{y-1}$, $y(1) = 1$ — $\partial f/\partial y$ discont. at $y = 1 \Rightarrow$ unique not guaranteed

(c). $(9 - t^2)y' + t(3+t)y = 12 + 4t$, $y(2) = 5$ — $-\infty < t < 3$

4. Be able to sketch and interpret a direction field.

5. Sketch the phase line, identify and classify the equilibrium solutions as stable, unstable, or semistable for the following differential equations. Determine behavior as $t \to \infty$ for the specified initial condition.

(a). $\dfrac{dp}{dt} = p(p - k_1)(k_2 - p), p(0) = p_0$ where $0 < \mathrm{k}_1 < \mathrm{p}_0 < \mathrm{k}_2$ — $p = 0$: stable sink; $p = k_1$: unstable source; $p = k_2$: stable sink; For IC $k_1 < p_0 < k_2, p(t) \to k_2$ as $t \to \infty$.

(b). $y' = \sin^2 y$, $y(0) = \pi/2$ — $y = \pm n\pi$ are all semistable nodes. For the given IC, $y(t) \to \pi$ as $t \to \infty$

6. Determine for which values of m the function $\phi(x) = x^m$ is a solutions to $3x^2 \dfrac{d^2y}{dx^2} + 11x\dfrac{dy}{dx} - 3y = 0$.

$m = \frac{1}{3}, -3$

7. Solve the following equations and initial value problems. Use any technique from class.

(a). $\dfrac{dy}{dx} = 2\sqrt{y+1}\cos x, \quad y(\pi) = 0.$ $\sqrt{y+1} = \sin x + 1 \Rightarrow y = (\sin x + 1)^2 - 1$

(b). $\dfrac{dx}{dt}x^2 = x$ $x^2 = 2t + C$

(c). $x^2\dfrac{dy}{dx} + \cos x = y$ $y(x) = -e^{-\frac{1}{x}} \int e^{\frac{1}{x}} \dfrac{\cos x}{x^2}\, dx + Ce^{-\frac{1}{x}}$ or $y(x) = -e^{-\frac{1}{x}} \int_a^x e^{\frac{1}{t}} \dfrac{\cos t}{t^2}\, dt + Ce^{-\frac{1}{x}}$

(d). $(x^2+1)\dfrac{dy}{dx} + xy = x$ $y = 1 + \dfrac{C}{\sqrt{x^2+1}}$

(e). $\dfrac{dy}{dx} + \dfrac{3y}{x} + 2 = 3x, \quad y(1) = 1$ $y(x) = \dfrac{3}{5}x^2 - \dfrac{1}{2}x + \dfrac{9}{10}x^{-3}$

(f). $(t/y)dy + (1 + \ln y)dt = 0$ $t\ln y + t + C = 0$

8. Han Solo and Princess Leia are trapped in a giant trash compacter that is 10 x 10 x 10 ft^3. In addition to the walls compressing together at a rate of 1 cubic foot per second, air containing .15% carbon monoxide is being pumped into the room at a rate of 2 ft^3/sec. It is well-mixed and leaves at the same rate. If the initial air in the compacter was "fresh" (at least in terms of carbon monoxide), how long do they have to quit bickering and escape if death occurs when the air in the room is .2% carbon monoxide. Which will kill them first, the carbon monoxide or the walls closing in? Note: $Q(t) = .003(1000-t) - 3\times10^{-6}(1000-t)^2$ So, $Q(t)/V(t) = .002$ when $t = 666.67$ s (i.e. 11.1 min.). Now, you need to argue when they would die from the crushing walls. For example, let's say that when the room is 4 x 4 x 4 ft^3, they will also die. So $V(t) = 64$ when $t = 936$ s (or 15.6 min). In this case, the carbon monoxide kills them first.

9. Section 2.3, #17

10. Other applications/homework from 2.3 and 2.5