

1. Classify the following differential equation as ordinary (ODE) or partial (PDE). Determine the order and state whether it is linear or nonlinear. Also, indicate the independent and dependent variables.

$$\frac{d^3y}{dt^3} + \cos t \frac{dy}{dt} + y = t$$

2. Determine whether the given function (explicit solutions) or relation (implicit solutions) is a solution to the given differential equation.

(a).  $y = 3 \sin 2x + e^{-x}$ ,  $y'' + 4y = 5e^{-x}$

(b).  $x^2 - \sin(x + y) = 1$ ,  $\frac{dy}{dx} = 2x \sec(x + y) - 1$

3. Determine whether the following initial value problems have a unique solution. If so, what is the largest interval or rectangle on which a unique solution exists.

(a).  $\frac{dy}{dx} = 3x - \sqrt[3]{y-1}$ ,  $y(1) = 2$

(b).  $\frac{dy}{dx} = 3x - \sqrt[3]{y-1}$ ,  $y(1) = 1$

(c).  $(9 - t^2)y' + t(3 + t)y = 12 + 4t$ ,  $y(2) = 5$

4. Be able to sketch and interpret a direction field.

5. Sketch the phase line, identify and classify the equilibrium solutions as stable, unstable, or semistable for the following differential equations. Determine behavior as  $t \rightarrow \infty$  for the specified initial condition.

(a).  $\frac{dp}{dt} = p(p - k_1)(k_2 - p)$ ,  $p(0) = p_0$  where  $0 < k_1 < p_0 < k_2$

(b).  $y' = \sin^2 y$ ,  $y(0) = \pi/2$

6. Determine for which values of  $m$  the function  $\phi(x) = x^m$  is a solutions to  $3x^2 \frac{d^2y}{dx^2} + 11x \frac{dy}{dx} - 3y = 0$ .

7. Solve the following equations and initial value problems. Use any technique from class.

(a).  $\frac{dy}{dx} = 2\sqrt{y+1} \cos x$ ,  $y(\pi) = 0$ .

(b).  $\frac{dx}{dt} x^2 = x$

(c).  $x^2 \frac{dy}{dx} + \cos x = y$

(d).  $(x^2 + 1) \frac{dy}{dx} + xy = x$

(e).  $\frac{dy}{dx} + \frac{3y}{x} + 2 = 3x$ ,  $y(1) = 1$

(f).  $(t/y)dy + (1 + \ln y)dt = 0$

8. Han Solo and Princess Leia are trapped in a giant trash compacter that is  $10 \times 10 \times 10 \text{ ft}^3$ . In addition to the walls compressing together at a rate of 1 cubic foot per second, air containing .15% carbon monoxide is being pumped into the room at a rate of  $2 \text{ ft}^3/\text{sec}$ . It is well-mixed and leaves at the same rate. If the initial air in the compacter was “fresh” (at least in terms of carbon monoxide), how long do they have to quit bickering and escape if death occurs when the air in the room is .2% carbon monoxide. Which will kill them first, the carbon monoxide or the walls closing in?

9. Section 2.3, #17

10. Other applications/homework from 2.3 and 2.5