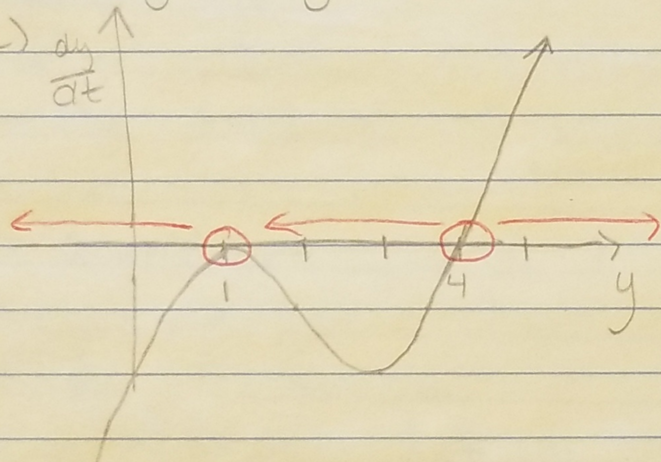
1. $\frac{dy}{dx} = \frac{2y(3-x)}{x(3-4y)}$

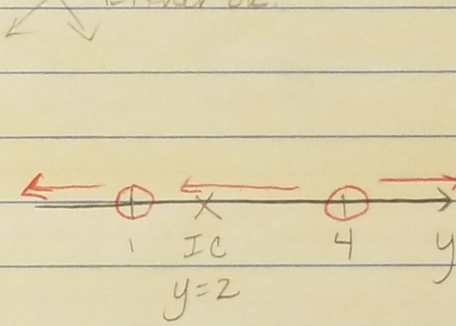
$\boxed{\text{ODE; Nonlinear; } 1^{st} \text{ order} \quad \text{Dep. Var.: } y \quad \text{Indep. Var.: } x}$

2. $\frac{dy}{dt} = (y-1)^2(y-4)$

(a)



Either ok.



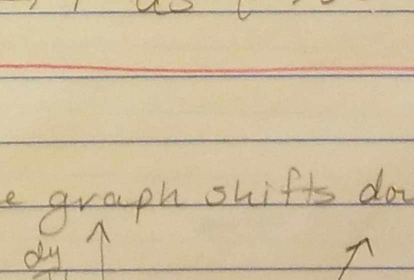
(b) $\boxed{\begin{array}{l} y=1 \text{ is semistable} \\ y=4 \text{ is unstable} \end{array}}$

(c) If $y(0)=2$, then $\boxed{y(t) \to 1 \text{ as } t \to \infty}$

(d) If $\frac{dy}{dt} = (y-1)^2(y-4) - 0.5$, the graph shifts down.

For $y(0)=2$, the sol$^{\underline{n}}$

$\boxed{y(t) \to -\infty \text{ as } t \to \infty}$



3(a) $ty' + 2y = \frac{1}{t^2}$

$y' + \frac{2}{t}y = \frac{1}{t^3}$ $\qquad \mu = e^{\int \frac{2}{t}dt} = e^{2\ln t} = e^{\ln t^2} = t^2$

$\Rightarrow$

$\underbrace{t^2 y' + 2ty}_{} = \frac{1}{t}$

$\frac{d}{dt}[t^2 y] = \frac{1}{t}$

$t^2 y = \int \frac{1}{t}\,dt$

$t^2 y = \ln|t| + C \Rightarrow \boxed{y(t) = \frac{1}{t^2}\ln|t| + \frac{C}{t^2}}$

(b) $\frac{dy}{dt} = 2(y+1)^2 \tan t \quad , \; y(0) = 1$

$\frac{1}{(y+1)^2}\,dy = 2\tan t\,dt$

$\int \frac{1}{(y+1)^2}\,dy = \int 2\tan t\,dt$

$u = y+1$ $\Rightarrow \int \frac{1}{u^2}\,du = \int 2\tan t\,dt \Rightarrow \int u^{-2}\,du = \int 2\tan t\,dt$

$du = dy$

$-\frac{1}{u} = 2\ln|\sec t| + C$

$-\frac{1}{y+1} = 2\ln|\sec t| + C$

IC $\quad -\frac{1}{1+1} = 2\ln|\sec 0| + C$

$-\frac{1}{2} = 2 \cdot \underbrace{\ln(1)}_{0} + C \Rightarrow C = -\frac{1}{2}$

$\boxed{-\frac{1}{y+1} = 2\ln|\sec t| - \frac{1}{2}}$ Implicit Form

$y+1 = \dfrac{1}{\frac{1}{2} - 2\ln|\sec t|}$

$y(t) = \dfrac{1}{\frac{1}{2} - 2\ln|\sec t|} - 1$ ← Explicit

$= \dfrac{1 - (\frac{1}{2} - 2\ln|\sec t|)}{\frac{1}{2} - 2\ln|\sec t|}$ Simplified.

$= \dfrac{\frac{1}{2} + 2\ln|\sec t|}{\frac{1}{2} - 2\ln|\sec t|} = \dfrac{1 + 4\ln|\sec t|}{1 - 4\ln|\sec t|}$

4. $e^x(y+x)\,dx + (y^2+e^x)\,dy = 0$

$M_y = e^x \qquad N_x = e^x \checkmark$ Exact

$\frac{\partial \psi}{\partial x} = e^x(y+x) = e^x y + xe^x$

$\psi(x,y) = \int ye^x + xe^x \, dx$

$= ye^x + xe^x - e^x + g(y)$

Diff u	Int dv
$+\ x$	e^x
$-\ 1$	e^x
$+\ 0$	e^x

OR use Integration Table

$\frac{\partial \psi}{\partial y} = \cancel{e^x} + g'(y) = y^2 + \cancel{e^x}$

$g'(y) = y^2$

$g(y) = \int y^2 dy$

$= \frac{1}{3}y^3 + \cancel{C_0}$

$\psi(x,y) = ye^x + xe^x - e^x + \frac{1}{3}y^3$

Sol^n: $\boxed{ye^x + xe^x - e^x + \frac{1}{3}y^3 = C}$

5. (a) $r_i = 10$ L/min $\qquad r_o = 10.2$ L/min

$C_i = 3$ kg/L $\qquad C_o = \frac{Q(t)}{V(t)} = \frac{Q(t)}{750-0.2t}$

$\frac{dQ}{dt} = r_iC_i - r_oC_o = 10\cdot 3 - 10.2\,\frac{Q(t)}{750-0.2t}$

$\Rightarrow \boxed{\begin{array}{c}\frac{dQ}{dt} = 30 - \frac{10.2}{750-0.2t}Q \\ Q(0) = 25\end{array}}$

(b) Valid until the tank empties $\Rightarrow V(t) = 0 \Rightarrow 750 - 0.2t = 0$

$\Rightarrow t = \frac{750}{0.2} = \boxed{3750 \text{ min}}$

6. $y' + p(t)y = 0 \qquad y(0) = 1$

where $p(t) = \begin{cases} 2, & 0 \le t \le 1 \\ 1, & t > 1 \end{cases}$

(a) For $0 \le t \le 1$ ← $t=0$

$y' + 2y = 0, \quad y(0) = 1$

$\mu = e^{\int 2dt} = e^{2t}$

$\underbrace{e^{2t}y' + 2e^{2t}y} = 0 \cdot e^{2t}$

$\frac{d}{dt}\left[e^{2t}y\right] = 0$

$e^{2t}y = \int 0\,dt$

$e^{2t}y = C$

$y(t) = Ce^{-2t}$ on $0 \le t \le 1$

$y(0) = Ce^{0} = 1$

$C = 1$

$\boxed{y_1(t) = e^{-2t} \text{ on } 0 \le t \le 1}$

For $t > 1$

$y' + y = 0$

$\mu = e^{\int 1 dt} = e^{t}$

$\underbrace{e^{t}y' + e^{t}y} = e^{t} \cdot 0$

$\frac{d}{dt}\left[e^{t}y\right] = 0$

$e^{t}y = \int 0\,dt$

$e^{t}y = C$

$\boxed{y_2(t) = Ce^{-t} \text{ on } t > 1}$

(b) At $t = 1 \Rightarrow y_1(1) = e^{-2}$ and $y_2(1) = Ce^{-1}$

Set equal: $e^{-2} = Ce^{-1}$

$e^{1} \cdot e^{-2} = C$

$\boxed{e^{-1} = C}$

$\Rightarrow \boxed{y_2(t) = e^{-1}e^{-t} = e^{-(t+1)}}$

(c) $\boxed{y(t) = \begin{cases} e^{-2t}, & 0 \le t \le 1 \\ e^{-(t+1)}, & t > 1 \end{cases}}$