Name: ________________________________ **Exam 1**

Math 341 Differential Equations – Crawford **07 March 2019**

	Score
1	/8
2	/14
3	/28
4	/16
5	/14
6	/22
Total	/100

Books and notes are not allowed. You may use a calculator and an integral table. ***Show all your work*** – partial credit may be given for written work.

Put all work and answers on the separately provided paper. Staple the exam on top.

Good Luck!

Calculator Number:

Formulas that may or may not be helpful

$$m\frac{dv}{dt} = mg - bv,\ v(0) = v_0 \implies v(t) = \frac{mg}{b} + \left(v_0 - \frac{mg}{b}\right)e^{-\frac{b}{m}t} \quad \text{and } x(0) = x_0 \implies x(t) = \frac{mg}{b}t + \frac{m}{b}\left(v_0 - \frac{mg}{b}\right)\left(1 - e^{-\frac{b}{m}t}\right) + x_0$$

$$\frac{dP}{dt} = -aP(P-K),\quad P(0) = P_0 \implies P(t) = \frac{P_0 K}{P_0 + (K - P_0)e^{-aKt}}$$

1. (8 pts). Classify the following differential equation as an ordinary or partial differential equation (ODE or PDE) and indicate whether it is linear or nonlinear. Give the order, and clearly indicate the independent variable(s) and dependent variable(s).

$$\frac{dy}{dx} = \frac{2y(3-x)}{x(3-4y)}$$

2. (14 pts). Given the differential equation $\frac{dy}{dt} = (y-1)^2(y-4)$,

(a). Sketch the phase line.

(b). Determine the equilibrium points and classify each one as stable, unstable, or semistable.

(c). If the initial condition is $y(0) = 2$, what will happen to the solution as $t \to \infty$?

(d). If the differential equation is changed to be $\frac{dy}{dt} = (y-1)^2(y-4) - 0.5$ and the initial condition is $y(0) = 2$, what will happen to the solution as $t \to \infty$?

3. (28 pts). Solve the following differential equations and initial value problems.

(a). $ty' + 2y = \dfrac{1}{t^2}$ [Write the final answer in explicit form.]

(b). $\dfrac{dy}{dt} = 2(y+1)^2 \tan t, \qquad y(0) = 1$ [Leave the answer in implicit form.]

4. (16 pts). Verify that the following differential equation is exact. Then find the solution.

$$e^x(y+x)\,dx + (y^2 + e^x)\,dy = 0$$

5. (14 pts). Suppose a brine mixture of 3 kg salt per liter runs into a tank initially filled with 750 L of water containing 25 kg of salt. The brine enters the tank at a rate of 10 L/min. The solution is well-mixed in the tank and is pumped out at the same rate. However, there is a small leak in the tank and an additional 0.2 L/min of fluid flows out of the tank.

(a). Set up but do ***not*** solve the *initial value problem* for the amount $Q(t)$ of salt in the tank at time t.

(b). Over what time interval is your model in part (a) valid?

6. (22 pts). Consider the following linear differential equation with a discontinuous coefficient.

$$y' + p(t)\,y = 0, \qquad y(0) = 1,$$

where

$$p(t) = \begin{cases} 2, & 0 \le t \le 1 \\ 1, & t > 1 \end{cases}$$

(a). Solve the differential equation in each interval ($0 \le t \le 1$ and $t > 1$), using the initial condition appropriately.

(b). Match the two solutions found in part (a) so that the solution y is continuous at $t = 1$.

(c). Write your final answer in piecewise form i.e. $y(t) = \begin{cases} \qquad\qquad , & 0 \le t \le 1 \\ \qquad\qquad , & t > 1 \end{cases}.$