1. Determine whether the following initial value problem has a unique solution. If so, what is the largest interval or rectangle on which a unique solution exists.
$x(x-3)y'' + 2xy' - y = x^2, \quad y(1) = y_0, y'(1) = y_1$ On $(0,3)$

2. (11th Edition) Section 3.2, p. 119 #10 [Be sure to show that y_1 and y_2 are linearly independent.]

3. Write the form only of the particular solution y_p. Do not evaluate the coefficients.

(a). $y'' + 5y' + 25y = (1+x^3)e^{-5x}$ $\quad y_p(x) = (Ax^3 + Bx^2 + Cx + D)e^{-5x}$

(b). $y^{(v)} - y''' - 2y' = 8x\cos x + \sin x + 3$ $\quad y_p(x) = (Ax+B)x\cos x + (Cx+D)x\sin x + Ex$

4. Determine a particular solution to the following differential equations

(a). $y'' + y' - 12y = 3x - e^{-2x}$ $\quad y_p(x) = -\frac{1}{4}x - \frac{1}{48} + \frac{1}{10}e^{-2x}$

(b). $y'' + y = e^{3x}\cos x$ $\quad y_p(x) = \frac{1}{13}e^{3x}\cos x + \frac{2}{39}e^{3x}\sin x$

5. Given that $y_1(x) = \frac{1}{4}\sin 2x$ is a solution to $y'' + 2y' + 4y = \cos 2x$ and $y_2(x) = \frac{1}{4}x - \frac{1}{8}$ is a solution to $y'' + 2y' + 4y = x$, find solutions to $y'' + 2y' + 4y = 3x - 4\cos 2x$. $\quad y_p(x) = 3(\frac{1}{4}x - \frac{1}{8}) - 4\left(\frac{1}{4}\sin 2x\right) = \frac{3}{4}x - \frac{3}{8} - \sin 2x$

6. Solve the following equations and initial value problems

(a). $4y'' + 9y = 0, \quad y(0) = 3, y'(0) = 2$ $\quad y(t) = 3\cos\frac{3}{2}t + \frac{4}{3}\sin\frac{3}{2}t$

(b). $y'' - 2y' + 6y = 0$ $\quad y(t) = c_1e^t\cos\sqrt{5}t + c_2e^t\sin\sqrt{5}t$

(c). $y'' + y' - 12y = 0$ $\quad y(x) = C_1e^{-4x} + C_2e^{3x}$

(d). $y''' + 3y'' + 4y' + 12y = 0$ $\quad y(x) = c_1e^{-3} + c_2\cos 2x + c_3\sin 3x$

(e). $y'' - y' - 6y = 36x^2, \quad y(0) = 3, y'(0) = \frac{4}{3}$ $\quad y(x) = \frac{10}{3}e^{-2x} + 2e^{3x} - 6x^2 + 2x - \frac{7}{3}$

(f). $3y'' + 12y = 4\sin 3t$ $\quad y(t) = C_1\cos 2t + C_2\sin 2t - \frac{4}{15}\sin 3t$

(g). $y'' - 3y' + 2y = x(e^x + 1), \quad y(0) = 1, y'(0) = -1$ $\quad y(x) = -\frac{3}{4}e^{2x} + e^x - \frac{1}{2}x^2e^x - xe^x + \frac{1}{2}x + \frac{3}{4}$

(h). $D^3(D-2)^2(D^2+9)[y] = 0$ $\quad y(x) = c_1 + c_2x + c_3x^2 + c_4e^{2x} + c_5xe^{2x} + c_6\cos 3x + c_7\sin 3x$

7. Find a general solution to the following differential equations.

(a). $x^2y'' + 2xy' - 2y = x^{-1}$ given that $y_1 = \dfrac{1}{x^2}$ and $y_2 = x$ are solutions to the homogeneous equation.
$y(x) = C_1\dfrac{1}{x^2} + C_2x - \dfrac{1}{2}\cdot\dfrac{1}{x}$

(b). $y'' + y = \tan x$ $\quad y(x) = C_1\cos x + C_2\sin x - \cos x\ln|\sec x + \tan x|$

(c). $y'' + y = \tan x + e^{3x}\cos x$ [Hint: No work necessary. Use part **(b)** and # **4(b)**.]
$y(x) = C_1\cos x + C_2\sin x - \cos x\ln|\sec x + \tan x| + \dfrac{1}{13}e^{3x}\cos x + \dfrac{2}{39}e^{3x}\sin x$

8. Use reduction of order to find a 2nd linearly independent solution, given that y_1 is a solution to the differential equation. Also, give the general solution.

(a). $x^2y'' - x(x+2)y' + (x+2)y = 0$ and $y_1(x) = x$ $\quad y_2(x) = xe^x$ then $y(x) = C_1x + C_2xe^x$

(b). $x^2y'' + 2xy' = 0$ and $y_1(x) = 1$ $\quad y_2(x) = \frac{1}{x}$ then $y(x) = C_1 + C_2\frac{1}{x}$

9. (11th Edition) Section 3.7, p. 157: #5

10. A 1 kg mass is attached to a spring with stiffness 26 N/m. The damping constant is 2 N-sec/m. At $t = 0$ an external force of $82\cos 4t$ is applied. If the spring is initially stretched 6 m and given a positive velocity of 1 m/s, determine the equation of motion for the system and sketch the solution. Write the steady-state solution in the form $R\cos(\omega t - \delta)$

$y(t) = e^{-t}\cos 5t\frac{14}{5} + e^{-t}\sin 5t + 5\cos 4t + 4\sin 4t.$ $\quad$ Steady State Soln $= y_p(t) = 5\cos 4t + 4\sin 4t = \sqrt{41}\cos(4t - 0.6747)$

11. If an undamped spring-mass system with a mass that weighs 6 lb and a spring constant 1 lb/in is suddenly set in motion at $t = 0$ by an external force of $4\cos 7t$ lb, determine the position of the mass at any time and draw a graph of the solution. $\quad u(t) = \frac{64}{45}(\cos 7t - \cos 8t) = \frac{128}{45}\sin\left(\frac{1}{2}t\right)\sin\left(\frac{15}{2}t\right)$

12. Solve the following differential equations and initial value problems.

(a). $x^2y'' + 2xy' - 6y = 0$ $\quad y(x) = C_1x^2 + C_2\frac{1}{x^3}$

(b). $2x^2y'' + 2xy' + 8y = 0, \quad y(1) = 0, y'(1) = -3$ $\quad y(x) = -\frac{3}{2}\sin(2\ln x)$