1. Determine whether the following initial value problem has a unique solution. If so, what is the largest interval or rectangle on which a unique solution exists.
$x(x-3)y'' + 2xy' - y = x^2, \;\; y(1) = y_0, y'(1) = y_1$

2. (11th Edition) Section 3.2, p. 119 #10 [Be sure to show that y_1 and y_2 are linearly independent.]

3. Write the form only of the particular solution y_p. Do not evaluate the coefficients.

(a). $y'' + 5y' + 25y = (1 + x^3)e^{-5x}$

(b). $y^{(v)} - y''' - 2y' = 8x\cos x + \sin x + 3$

4. Determine a particular solution to the following differential equations

(a). $y'' + y' - 12y = 3x - e^{-2x}$

(b). $y'' + y = e^{3x}\cos x$

5. Given that $y_1(x) = \frac{1}{4}\sin 2x$ is a solution to $y'' + 2y' + 4y = \cos 2x$ and $y_2(x) = \frac{1}{4}x - \frac{1}{8}$ is a solution to $y'' + 2y' + 4y = x$, find solutions to $y'' + 2y' + 4y = 3x - 4\cos 2x$.

6. Solve the following equations and initial value problems

(a). $4y'' + 9y = 0, \;\; y(0) = 3, y'(0) = 2$

(b). $y'' - 2y' + 6y = 0$

(c). $y'' + y' - 12y = 0$

(d). $y''' + 3y'' + 4y' + 12y = 0$

(e). $y'' - y' - 6y = 36x^2, \;\; y(0) = 3, y'(0) = \frac{4}{3}$

(f). $3y'' + 12y = 4\sin 3t$

(g). $y'' - 3y' + 2y = x(e^x + 1), \;\; y(0) = 1, y'(0) = -1$

(h). $D^3(D-2)^2(D^2+9)[y] = 0$

7. Find a general solution to the following differential equations.

(a). $x^2y'' + 2xy' - 2y = x^{-1}$ given that $y_1 = \dfrac{1}{x^2}$ and $y_2 = x$ are solutions to the homogeneous equation.

(b). $y'' + y = \tan x$

(c). $y'' + y = \tan x + e^{3x}\cos x$ [Hint: No work necessary. Use part **(b)** and # **4(b)**.]

8. Use reduction of order to find a 2nd linearly independent solution, given that y_1 is a solution to the differential equation. Also, give the general solution.

(a). $x^2y'' - x(x+2)y' + (x+2)y = 0$ and $y_1(x) = x$

(b). $x^2y'' + 2xy' = 0$ and $y_1(x) = 1$

9. (11th Edition) Section 3.7, p. 157: #5

10. A 1 kg mass is attached to a spring with stiffness 26 N/m. The damping constant is 2 N-sec/m. At $t = 0$ an external force of $82\cos 4t$ is applied. If the spring is initially stretched 6 m and given a positive velocity of 1 m/s, determine the equation of motion for the system and sketch the solution. Write the steady-state solution in the form $R\cos(\omega t - \delta)$

11. If an undamped spring-mass system with a mass that weighs 6 lb and a spring constant 1 lb/in is suddenly set in motion at $t = 0$ by an external force of $4\cos 7t$ lb, determine the position of the mass at any time and draw a graph of the solution.

12. Solve the following differential equations and initial value problems.

(a). $x^2y'' + 2xy' - 6y = 0$

(b). $2x^2y'' + 2xy' + 8y = 0, \quad y(1) = 0, y'(1) = -3$