Axioms of Neutral Geometry

1. The Existence Postulate

2. The Incidence Postulate

3. The Ruler Postulate

4. The Plane Separation Postulate

5. The Protractor Postulate

6. The Side-Angle-Side Postulate

Add the Euclidean Parallel Postulate and we finally get to Euclidean Geometry.

Axiom 7 Euclidean Parallel Postulate For every line l and for every point P that does not lie on l, there is exactly one line m such that P lies on m and $m \parallel l$.

Prove each of the following in Euclidean Geometry. [Note: They are not true in Neutral or Hyperbolic Geometry.]

1. Converse to the Alternate Interior Angles Theorem : If two parallel lines are cut by a transversal then both pairs of alternating interior angles are congruent.

Sketch a diagram.

Hint: Use proof by contradiction.

Proof

2. 180 DEGREES THEOREM : For every triangle, the interior angle sum is 180°.

[Hint: Draw a line parallel to one side through the opposite vertex.]

PROOF

3. PROCLUS' AXIOM : If l and m are parallel lines and t is a line such that $t \neq l$ and t intersects l, then t also intersects m.

[Sketch]

PROOF

4. Perpendicular Transversal Theorem : If l and m are parallel lines and t is a transversal such that $t \perp l$ then $t \perp m$.

[Sketch]

Proof

5. Transitivity of Parallelism : If $l \parallel m$ and $m \parallel n$ and $l \neq n$ or $l \parallel n$.

[Sketch]

Proof