

AXIOMS OF NEUTRAL GEOMETRY

1. The Existence Postulate
2. The Incidence Postulate
3. The Ruler Postulate
4. The Plane Separation Postulate
5. The Protractor Postulate
6. The Side-Angle-Side Postulate

“Neutral” because Neutral Geometry \_\_\_\_\_ :

EUCLIDEAN PARALLEL POSTULATE For every line  $l$  and for every point  $P$  that does not lie on  $l$ , there is exactly one line  $m$  such that  $P$  lies on  $m$  and  $m \parallel l$ .

ELLIPTIC PARALLEL POSTULATE For every line  $l$  and for every point  $P$  that does not lie on  $l$ , there no line  $m$  such that  $P$  lies on  $m$  and  $m \parallel l$ .

HYPERBOLIC PARALLEL POSTULATE For every line  $l$  and for every point  $P$  that does not lie on  $l$ , there are at least two lines  $m$  and  $n$  such that  $P$  lies on both  $m$  and  $n$  and both  $m$  and  $n$  are parallel to  $l$ .

THEOREM 4.9.1 (The Universal Hyperbolic Theorem)

If there exists one line  $l_0$ , an external point  $P_0$ , and at least two lines that pass through  $P_0$  which are parallel to  $l_0$ , then for \_\_\_\_\_ line  $l$  and for \_\_\_\_\_ external point  $P$ , there exist \_\_\_\_\_ that pass through  $P$  and are parallel to  $l$ .

COROLLARY 4.9.2 The Hyperbolic Parallel Postulate is equivalent to the \_\_\_\_\_ of the Euclidean Parallel Postulate.

COROLLARY 4.9.3 In any model for \_\_\_\_\_ either the Euclidean Parallel Postulate or the Hyperbolic Parallel Postulate will hold.

In the presence of the 6 Neutral Geometry Axioms,

- Many statements are logically equivalent to the \_\_\_\_\_
  - i.e. If the Euclidean Parallel Postulate is assumed as Axiom 7,  
then the other logically equivalent statements \_\_\_\_\_ .
  - or vice versa: If one of the other logically equivalent statements is assumed as Axiom 7,  
then the Euclidean Parallel Postulate \_\_\_\_\_ .
- Notably: \_\_\_\_\_ is logically equivalent to the Euclidean Parallel Postulate.

EUCLID'S POSTULATE V . That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

Sketch a diagram.

EUCLID'S POSTULATE V RESTATED .