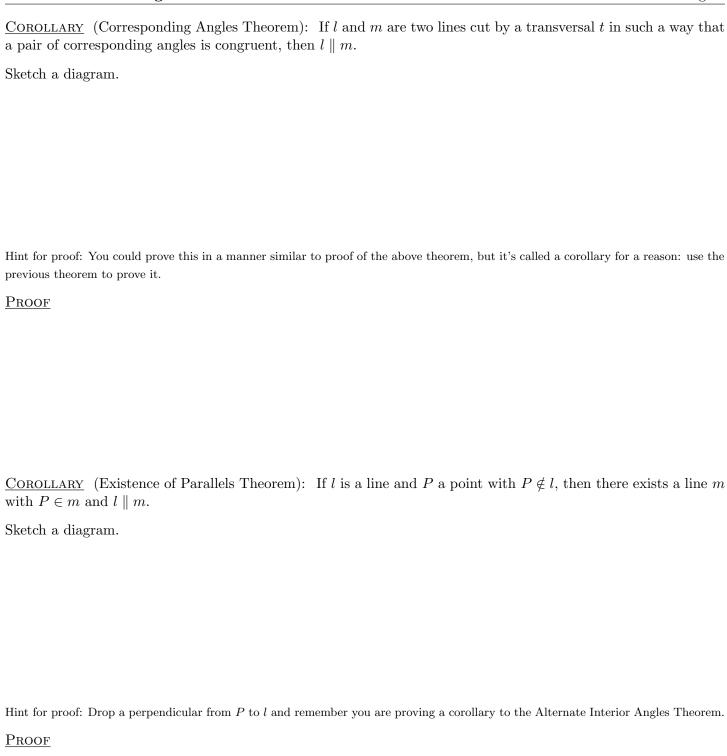
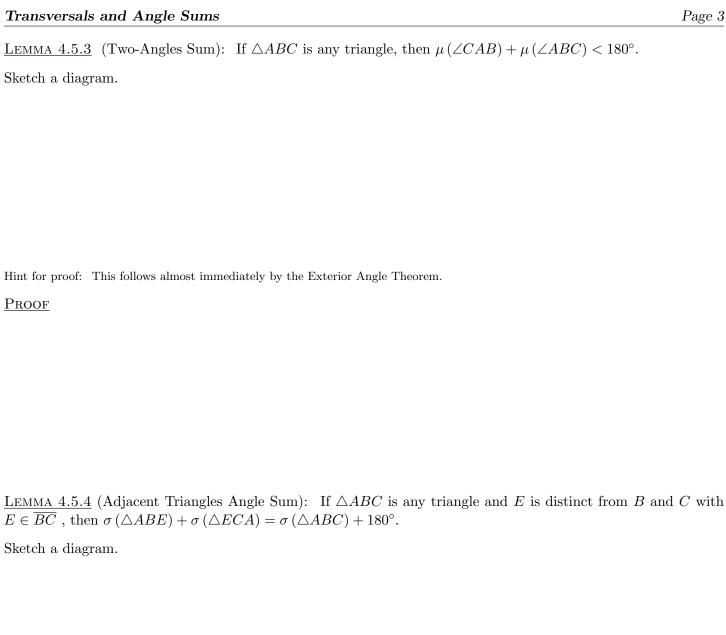
Theorem (Alternate Interior Angles Theorem): If $l$ and $m$ are two lines cut by a transversal $t$ in such a way that a pair of alternate interior angles is congruent, then $l \parallel m$ .
Sketch a diagram.
PROOF Let $l$ and $m$ be two lines cut by a transversal $t$ in such a way that a pair of alternate interior angles is congruent. [Show that $l \parallel m$ .]
Choose points $A, B, C$ and $A', B', C'$ as in the definitions of transversal and interior angles.
WLOG, let the congruent alternate interior angles be $\angle A'B'B$ and $\angle B'BC$ . i.e. $\angle A'B'B \cong \angle B'BC$
BWOC, suppose $l$ and $m$
Then $l$ and $m$ must Let $D$ be this point of
Case 1: $D$ lies on the same side of $t$ as $C$ . [Sketch a new picture.]
Then $\angle A'B'B$ is an for $\triangle BB'D$ .
Then $\mu(\angle A'B'B) > \mu(\angle B'BD)$ by the
But $\angle B'BD = \underline{\hspace{1cm}} \Rightarrow \mu(\angle A'B'B) > \mu(\underline{\hspace{1cm}}) \rightarrow \leftarrow \text{ since } \underline{\hspace{1cm}}.$
Case 2: $D$ lies on the same side of $t$ as $A$ .
Then by a similar argument $\mu(\angle B'BC) > \mu(\angle DB'B) = \underline{\hspace{1cm}} \rightarrow \leftarrow$ .

Both cases lead to a contradictions. Therefore,  $\_\_\_\_$  .





Proof

<u>LEMMA 4.5.5</u> (Equal Angle Sum): If  $\triangle ABC$  is any triangle then there exists a point  $D \notin \overrightarrow{AB}$  such that  $\sigma(\triangle ABD) = \sigma(\triangle ABC)$  and the measure of one of the interior angles of  $\triangle ABD$  is less than or equal to  $\frac{1}{2}\mu(\angle CAB)$ .

Sketch a diagram.

Hint for proof: Let M be the midpoint of  $\overline{BC}$  and let A\*M\*D with AM=MD.

Proof

These lemmas will all be used to prove the following theorem. Close your book and fill in the blank below. Then look up the statement of the theorem on p. 85.

<u>Theorem</u> (Saccheri-Legendre Theorem): If  $\triangle ABC$  is any triangle, then  $\sigma$  ( $\triangle ABC$ ) 180°.

Is the conclusion of this theorem what you expected?

If not, why do you think it is different? [See the paragraph on p. 85 above the S-L Theorem.]

Proof Done together as a class.

Homework: Finish the proofs on this worksheet. Read the text of Sections 4.4-4.5 and summarize the main points.