[Close your books.]

1. <u>Theorem</u> (ASA): If $\triangle ABC$ and $\triangle DEF$ are triangles such that $\angle CAB \cong \angle FDE, \overline{AB} \cong \overline{DE}$, $\angle ABC \cong \angle DEF$, then $\triangle ABC \cong \triangle DEF$.

Sketch a diagram for this theorem.

<u>Proof</u> Let $\triangle ABC$ and $\triangle DEF$ be defined as above. [Show $\triangle ABC \cong \triangle DEF$.]

[If we can show congruence of another side, then we can use ___________ .]

Since the length DF is a nonnegative number, then by the ____________________________________ , there exists a point G on $\overrightarrow{AC}$ such that $AG =$ __________.

Therefore $\overline{AG} \cong$ __________. [To finish the proof: Use SAS and eventually show $G = C$.]

Now $\overline{AG} \cong \overline{DF}$, $\angle CAB \cong \angle FDE$, and $\overline{AB} \cong \overline{DE}$, so by __________ , $\triangle ABG$ ________$\triangle DEF$.

$\Rightarrow \angle ABG \cong \angle DEF$. But $\angle DEF \cong \angle ABC$ (given in hypothesis). Therefore, $\angle ABC \cong$ _____________.

$\Rightarrow \overrightarrow{BC} = \overrightarrow{BG}$, by the Protractor Postulate, Part _______ .

But $\overrightarrow{BC}$ can intersect $\overleftrightarrow{AC}$ in at most one point. (Theorem 3.1.7)

Therefore ____________ and $\triangle ABC \cong \triangle DEF$.

2. Recall the Isosceles Triangle Theorem 3.6.5 (restatement): If $\triangle ABC$ is a triangle and $\overline{AB} \cong \overline{AC}$, then $\angle ABC \cong \angle ACB$

<u>Theorem</u> (Converse to the Isosceles Triangle Theorem): [State the converse, then check your answer on p. 74.]

Sketch a diagram for this theorem.

<u>Proof</u> [Homework, Section 4.2, p. 77 #1] [Hint: Use ASA.]

Sketch a generic triangle (preferably not right, isosceles, or equilateral).

<u>Def</u> Let $\triangle ABC$ be a triangle. The angles $\angle CAB, \angle ABC$, and $\angle BCA$ are called _____________ <u>angles</u> of the triangle.

On your picture above, extend the segments $\overline{AC}$ and $\overline{BC}$ to be rays $\overrightarrow{AC}$ and $\overrightarrow{BC}$. Label a point on the extension $\overrightarrow{AC}$ to be E and a point on the extension $\overrightarrow{BC}$ to be D.

What can you say about angles $\angle BCA$ and $\angle ACD$?

What can you say about angles $\angle ACB$ and $\angle BCE$?

<u>Def</u> Let $\triangle ABC$ be a triangle. An angle that forms a linear pair with one of the interior angles is called an _____________ angle for the triangle. If the exterior angle forms a ___________ pair with the interior angle at one vertex, then the interior angles at the other two vertices are called remote interior angles.

What are the exterior angles for $\triangle ABC$ and vertex C?

What are the remote interior angles?

3. <u>THEOREM</u> (Exterior Angle Theorem). The measure of an exterior angle for a triangle is strictly greater than the measure of either remote interior angle.

Fill in the blanks to restate this theorem.

<u>RESTATEMENT</u>: Let $\triangle ABC$ be any triangle and let D lie on $\overleftrightarrow{BC}$ so that $\angle ACD$ is an exterior angle of the triangle. Then $\mu(\angle DCA) >$ _______________ and $\mu(\angle DCA) >$ _______________.

<u>PROOF</u> Let $\triangle ABC$ be any triangle and let D lie on $\overleftrightarrow{BC}$ so that $\angle ACD$ is an exterior angle of the triangle.

Case 1: [Show $\mu(\angle DCA) > \mu(\angle BAC)$]

Let E be the midpoint of $\overline{AC}$. (Existence of unique midpoint.)

Construct the point F so that E is the midpoint of $\overline{BF}$. (Point Construction Postulate.)

[Sketch a picture. Update the picture as you work through the proof.]

Thus, $\angle BEA$ and $\angle FEC$ form a _____________ pair and are therefore _______________ by the Vertical Angles Theorem.

Thus, $\triangle BEA \cong$ _____________ by SAS.

So $\angle BAC \cong$ ___________, by definition of congruent triangles. (*)

By construction, F is on the interior of $\angle DCA$.

Thus, $\mu(\angle FCA) < \mu(\angle DCA)$ by Betweenness for Rays. (**)

By (*) and (**), we have $\mu(\angle \qquad) < \mu(\angle DCA)$.

Case 2: [Show $\mu(\angle DCA) > \mu(\angle ABC)$] [Finish the proof as homework. Hint: Extend the side $\overline{AC}$ to create an angle congruent to $\angle ACD$.]

<u>Homework</u>

Finish the worksheet, including Section 4.2, p. 77 #1 and Case 2 above. Section 4.1, p. 73 #1
Sketch two triangles that show that SSA is not a valid triangle congruence condition. i.e. If you have congruence of SSA, it does not guarantee that the two triangles will be congruent.