[Close your books.]

1. Theorem (The Z-Theorem): Let l be a line and let A and D be distinct points on l. If B and E are points on opposite sides of l, then $\overrightarrow{AB} \cap \overrightarrow{DE} =$ __$\emptyset$__.

Sketch a diagram for this theorem and fill in the blank above. [Can you see why it is called the Z-Theorem?]

Proof Let l be a line and let A and D be distinct points on l.

Also let B and E be on opposite sides of l.

By the (contrapositive of) the Ray Theorem, all points on $\overrightarrow{AB}$, except __A__ lie in __one half-plane__ determined by l,

Similarly, all points on $\overrightarrow{DE}$, except __D__ lie in __the other half-plane__ determined by l.

The half-planes do not intersect by the __Plane Separation__ Postulate.

Thus the only place the rays could intersect would be at __the endpoints__ .

But since A and D are __distinct__ , $\overrightarrow{AB} \cap \overrightarrow{DE} = \emptyset$.

2. Theorem (The Crossbar Theorem): Let $\triangle ABC$ be a triangle. If a point D is in the interior of $\angle BAC$, then $\overrightarrow{AD} \cap \overline{BC}$ __$\neq$__ $\emptyset$.

Sketch a diagram for this theorem and fill in the blank above. [Can you see why it is called the Crossbar Theorem?]

Fill in the blanks to (informally) restate the Crossbar Theorem: If a ray is in the __interior__ of one of the angles of a triangle, then the ray must intersect the __opposite__ side of the triangle.

Proof We'll do it later as a class.

3. <u>Theorem</u> A point D is in the interior of $\angle BAC$ if and only if $\overrightarrow{AD} \cap \overline{BC}$ $\underline{\quad \neq \quad}$ $\emptyset$.

Sketch a diagram for this theorem and fill in the blank above.

<u>Proof</u>

$\Rightarrow$: Let D be a point in the interior of $\angle BAC$. Then $\overrightarrow{AD}$ intersects $\overline{BC}$ by the <u>Crossbar Theorem</u> .

$\Leftarrow$: Let $\overrightarrow{AD} \cap \overline{BC} \neq \emptyset$.

Then let $E \in \overrightarrow{AD} \cap \overline{BC}$. Note that $\overrightarrow{AD} \underline{\quad = \quad} \overrightarrow{AE}$.

Then $B * E * C$. [How do you know that E is not B or C?]

Thus, by Theorem 3.3.10, $\underline{\quad \overrightarrow{AB} \quad} * \underline{\quad \overrightarrow{AE} \quad} * \underline{\quad \overrightarrow{AC} \quad}$.

Thus, E is in the interior of $\angle \underline{\quad BAC \quad}$

Since $D \in \overrightarrow{AE}$, D is in the interior of $\angle BAC$ by the Ray Theorem.

4. <u>Lemma</u> If $C * A * B$ and D is in the interior of $\angle BAE$ then E is in the interior of $\angle DAC$.

Sketch a diagram for this lemma.

<u>Proof</u> Let $C * A * B$ and let D be in the interior of $\angle BAE$.

Since D is in the interior of $\angle BAE$, D and E are on the <u>same side</u> of $\overleftrightarrow{AB}$.

But since $\underline{\quad \overleftrightarrow{AB} \quad} = \overleftrightarrow{AC}$, D and E are on the same side of $\underline{\quad \overleftrightarrow{AC} \quad}$.

By the Crossbar Theorem, $\underline{\quad \overrightarrow{AD} \cap \overline{BE} \neq \emptyset \quad}$.

Therefore E and B are on opposite sides of $\underline{\quad \overleftrightarrow{AD} \quad}$.

Since $C * A * B$, C and B are on <u>opposite sides</u> of $\overleftrightarrow{AD}$.

Thus C and E are on the <u>same side</u> of $\overleftrightarrow{AD}$ by the Plane Separation Postulate.

Therefore E is in the interior of $\angle DAC$.

5. <u>Theorem</u> (The Linear Pair Theorem): If angles $\angle BAD$ and $\angle DAC$ form a linear pair, then $\mu(\angle BAD) + \mu(\angle DAC) = \underline{\quad 180 \quad}^\circ$.

Sketch a diagram for this theorem and fill in the blank above.

<u>Proof</u> We'll do it later as a class.