[Close your books.]

<u>Axiom 4 (The Plane Separation Postulate)</u>. For every line l, the points that do not lie on l form two disjoint, nonempty sets H_1 and H_2, called half-planes bounded by l, such that

1. H_1 and H_2 are convex.

2. If $P \in H_1$ and $Q \in H_2$, then $\overline{PQ}$ intersects l.

The postulate can be restated using set theory notation. Fill in the following blanks for what the postulate says about H_1 and H_2:

- H_1________$H_2 = \mathbb{P} \setminus l$ Intersection or union?
- H_1________$H_2 = \emptyset$ Intersection or union?
- H_1________$\emptyset$ and H_2________$\emptyset$
- If $A \in H_1$ and $B \in H_1$, then $\overline{AB} \subseteq$ _________ and $\overline{AB} \cap l$________$\emptyset$
- If $A \in H_2$ and $B \in H_2$, then $\overline{AB} \subseteq$ _________ and $\overline{AB} \cap l$________$\emptyset$
- If $A \in H_1$ and $B \in H_2$, then $\overline{AB} \cap l$________$\emptyset$

1. <u>Theorem</u> (The Ray Theorem): Let l be a line, A a point on l, and B be an external point for l. If C is a point on $\overrightarrow{AB}$ and $C \neq A$, then B and C are on the same side of l

1. Sketch a diagram for this theorem.

2. <u>Proof</u> Let l be a line, A a point on l, and B an external point for l. Let C be a point on $\overrightarrow{AB}$ such that $C \neq A$. [Show that B and C are on the same side of l. i.e. Show that $\overline{BC} \cap l =$ ________.]

Note that A, B, and C are ______________ and so we the following three cases.

<u>Case 1 ($C = B$)</u>: Trivially true since B and C are the ___________ point, they are clearly on the same side of l.

<u>Case 2 ($A * C * B$)</u>: Then A is _________________ B and C since for 3 collinear points only one point is between the other two (previous corollary).

Thus A________$\overline{BC}$. (*)

Since B is not on l, the lines $\overleftrightarrow{AB}$ and l are _____________ .

Since A is on both l and $\overleftrightarrow{AB}$, they are not parallel.

Therefore, the two lines intersect _______________ one point (previous theorem), which we already know is point A.

But this point A is not in __________ from (*).

Therefore $\overline{BC} \cap l =$ _______ and B and C are on the same side of l.

<u>Case 3 ($A * B * C$)</u>: [Complete the proof.]

2. <u>THEOREM</u> Let A, B, and C be three noncollinear points and let D be a point on the line $\overleftrightarrow{BC}$. Then $B * D * C$ iff $\overrightarrow{AB} * \overrightarrow{AD} * \overrightarrow{AC}$

1. Sketch a diagram for this theorem.

2. <u>PROOF</u> Let A, B, and C be three noncollinear points and let D be a point on the line $\overleftrightarrow{BC}$.

$\Rightarrow$: Let $B * D * C$ [Show that $\overrightarrow{AB} * \overrightarrow{AD} * \overrightarrow{AC}$]

Then $D \in \overrightarrow{BC}$.

Then by the Ray Theorem, D and C are on the same side of __________ .

Similarly D and B are on the same side of $\overleftrightarrow{AC}$.

So D is in the half-plane determined by $\overleftrightarrow{AB}$ and containing C.

The point D is also in the half-plane determined by $\overleftrightarrow{AC}$ and containing B.

Therefore D is in the intersection of these two half-planes.

Then D is in the _____________ of angle _____________ by definition.

Therefore $\overrightarrow{AB} * \overrightarrow{AD} * \overrightarrow{AC}$ by definition of _________________ for rays.

$\Leftarrow$: Let $\overrightarrow{AB} * \overrightarrow{AD} * \overrightarrow{AC}$ [Show $B * D * C$]

Then D is in the interior of $\angle BAC$.

Therefore D is in the half-plane for $\overleftrightarrow{AB}$ that contains C. i.e. C and D are on _____________________ $\overleftrightarrow{AB}$.

Since the half-planes and their lines are disjoint sets (Plane Separation Postulate), the point B is not on the segment $\overline{CD}$. (*)

Similarly, C is not on the line segment __________ . (**)

Recall that the point D is on the line __________ (given in the hypothesis). Therefore B, C, and D are ______________ .

But by (*), B is not between C and D and by (**), C is not between B and D.

Therefore, D must be between ______________ . ■

3. <u>THEOREM</u> (Pasch's Theorem [Axiom]) Let $\triangle ABC$ be a triangle and let l be a line such that none of A, B, and C lies on l. If l intersects $\overline{AB}$, then l also intersects either $\overline{AC}$ or $\overline{BC}$.

1. Sketch a diagram for this theorem.

2. <u>PROOF</u> Let $\triangle ABC$ be a triangle and let l be a line such that none of A, B, and C lies on l.

Also, let l intersect $\overline{AB}$ [Show ______________________]

Since $\overline{AB} \cap l$________$\emptyset$, the points A and B are on ____________ of l.

<u>Case 1 (C is on the same side of l as A)</u>: Then C and B are on ____________ of l.

Therefore $\overline{BC}$ intersects l (______________ Postulate).

<u>Case 2 (C is on the opposite side of l as A)</u>: Then ________ intersects l (Plane Separation Postulate).

■

Homework:

Section 3.3, p. 47: #1, 2, 3, 5