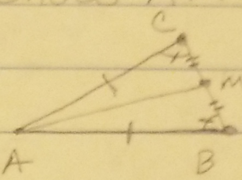
Take-Home Quiz 2.

1\. Proof: Let $\triangle ABC$ be a triangle with $AB = AC$.
Let M be the midpoint of $\overline{BC}$. [Show $\overleftrightarrow{AM} \perp \overleftrightarrow{BC}$]
Then $MC = MB$ by def. of midpoint



Also $\angle ABC \cong \angle ACB$ by the Isoceles Triangle Theorem.

$\Rightarrow \triangle ABM \cong \triangle ACM$ by SAS.

$\Rightarrow \angle AMB \cong \angle AMC$

But $\angle AMB$ and $\angle AMC$ form a linear pair.

$\Rightarrow \mu(\angle AMB) + \mu(\angle AMC) = 180^\circ$

Since these 2 angles are congruent, their measures are equal

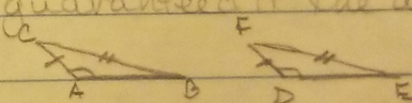
$\Rightarrow \mu(\angle AMB) = \mu(\angle AMC) = 90^\circ$

$\therefore \overleftrightarrow{AM} \perp \overleftrightarrow{BC}$ ∎

NOTE: Explanations <u>not</u> required nor graded.

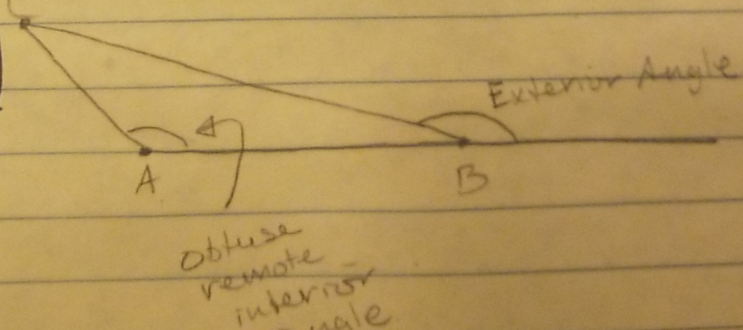
3\. (a) $\boxed{\text{TRUE}}$

(b) $\boxed{\text{FALSE}}$ Also guaranteed if the angle is obtuse



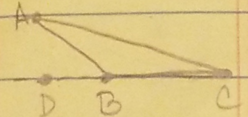
(c) $\boxed{\text{TRUE}}$ Use Point Construction & Theorem 4.2.6

(d) $\boxed{\text{FALSE}}$



Section 4.1, p. 73 #1

1. If one interior angle of a triangle is right or obtuse, then both the other interior angles are acute.



Proof Let $\triangle ABC$ be a triangle where (WLOG)
$\angle ABC$ is right or obtuse [Show $\angle BAC$ + $\angle BCA$ are acute]
ie $\mu(\angle ABC) \geq 90°$ (*) [Show $\mu(\angle BAC)$ + $\mu(\angle BCA) < 90°$]

Construct a pt D on $\overleftrightarrow{BC}$ s.t. $\angle ABD$ is an exterior angle at B.
Then by the Exterior Angle Theorem
$\mu(\angle ABD) > \mu(\angle BAC)$ and $\mu(\angle ABD) > \mu(\angle BCA)$ (**)

Also, by definition of exterior angle,
$\angle ABD$ forms a linear pair w/ $\angle ABC$.
Then by the Linear Pair Theorem
$\mu(\angle ABD) + \mu(\angle ABC) = 180°$
$\Rightarrow \mu(\angle ABD) = 180° - \mu(\angle ABC)$.
$\leq 90°$ since $\mu(\angle ABC) \geq 90°$ (from *)

So from (**) $\mu(\angle BAC) < \mu(\angle ABD) \leq 90°$
& $\mu(\angle BCA) < \mu(ABD) \leq 90°$

$\Rightarrow \mu(\angle BAC) < 90°$
and $\mu(\angle BCA) < 90°$

$\therefore$ Both angles are acute ∎