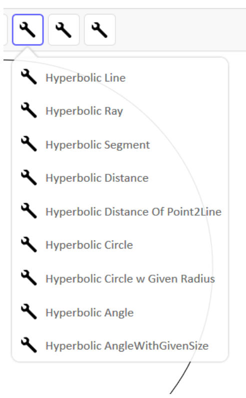
Hyperbolic Geometry Exploration

Go to the following link to open a GeoGebra application designed to explore Hyperbolic Geometry on the Poincaré Disk.

https://www.geogebra.org/m/R5e9AggU

You should see a GeoGebra window with a circular disk and a menu bar that looks like the following.

The three wrenches contain customized tools for exploring hyperbolic geometry. If you click on each one, a drop-down menu with the available tools appears:



Click on the other two wrenches to see the other available tools. Play around with a few of the tools to get comfortable with them.

Notice that the window to the right will show coordinates and measurements as you construct objects in the disk.

To clear your GeoGebra screen, click on the  icon in the upper right corner.

Vertical Angles:

- Construct two intersecting hyperbolic lines.
- Construct the point of intersection.
- Measure the vertical angles.
- Move the points around.

Does it appear that vertical angles are congruent in hyperbolic geometry?

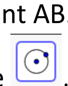
Linear Pairs:

- In the same construction as above, measure two angles in a linear pair.
- Compute the sum of angle measure of the linear pairs.
- Move the points around.

Does it appear that linear pairs are supplementary angles in hyperbolic geometry.

Perpendicular Bisector:

Construction 1:

- Construct a line segment AB.
- Using the regular circle , construct the circle with center at point A that passes through point B.

- Construct the regular circle with center at point B that passes through point A.
- Construct the two intersection points of the two circles. Let the two intersections points be labeled C and D. ***[Or leave them labeled as the points given by GeoGebra and adjust the instructions below accordingly.]***
- Construct line CD.
- Construct the intersection point of segment AB and line CD. Call the point E. ***[Or leave it labeled as the point given by GeoGebra and adjust the instructions below accordingly.]***

Does the line CD appear to bisect segment AB?

- Measure the lengths of AE and EB to confirm your guess.

So Construction 1 did or did not result in constructing the perpendicular bisector. [Circle one.]

[Clear the disk by clicking on]

Construction 2:

- Construct a line segment AB.
- Using the Hyperbolic Circle tool, construct the circle with center at point A that passes through point B.
- Construct the hyberbolic circle with center at point B that passes through point A.
- Construct the two intersection points of the two circles. Let the two intersections points be labeled C and D. ***[Or leave them labeled as the points given by GeoGebra and adjust the instructions below accordingly.]***
- Construct line CD.
- Construct the intersection point of segment AB and line CD. Call the point E. ***[Or leave it labeled as the point given by GeoGebra and adjust the instructions below accordingly.]***

Does the line CD appear to bisect segment AB?

- Measure the lengths of AE and EB to confirm your guess.
- Measure the angles to make sure they are 90 degrees.

[Clear the disk by clicking on]

Construction 3 [Which is probably Construction 2 built into a tool]:

- Construct a line segment AB.
- Use the Hyberbolic Perpendicular Bisector Tool to construct the perpendicular bisector to AB. [Select the tool and then select points A and B.]

[Clear the disk by clicking on]

Angle Sum for Triangles:

- Construct triangle ABC.
- Measure each angle.
- Compute the sum of the angles.
- Move the points around.

Does the sum of the angles equal 180 degrees?

Does the sum of the angles equal a constant value for all triangles?

Try to create a triangle whose angle sum is 180 degrees? What do you notice?

[Clear the disk by clicking on ]

<u>Isosceles Triangle:</u>

[Note: Your points may be labeled differently than the instructions below. Adjust the directions as needed.]

- Construct three non-collinear points A, B, and C.
- Construct a hyperbolic circle with center at A. But do not have the circle go through any of the other points.
- Construct the ray AB (with endpoint at A that passes through point B).
- Construct a ray AC (with endpoint at A that passes through point C).
- Let D be the intersection of ray AB and the circle with center at A.
- Let E be the intersection of ray AC and the circle with center at A.
- Construct triangle ADE. By construction, triangle ADE must be isosceles since two of its sides are radii of the same circle. Verify that the lengths are two sides are equal.
- Move the points around.

Do the base angles look congruent?

Measure the base angles. Does it confirm your guess?

[Clear the disk by clicking on ]

Additional Explorations:

1. Figure out how to construct an equilateral triangle. [Hint: Think about how the isosceles triangle was constructed.] Measure the lengths and angles and note any properties that are the same or different than for equilateral triangles in Euclidean Geometry.
2. Figure out how to construct a right triangle. [Hint: Look at your Hyperbolic Tools to see if you can create two lines or segments perpendicular to each other]. Measure the lengths of each side. Compute appropriate values to see whether the Pythagorean Theorem holds in Hyperbolic Geometry.