Name: Key

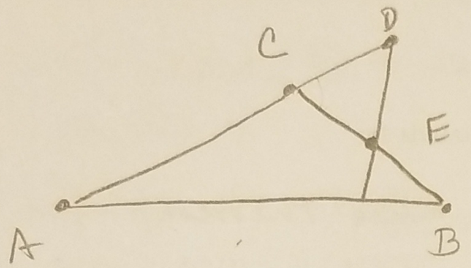
Math 331 Foundations of Geometry – Crawford

Exam 2

05 November 2018

Books and notes (in any form) are not allowed. You may use the provided list of Axioms, Postulates, and Theorems. ***Show all your work.*** Partial credit may be given for written work. Good Luck!

1. (16 pts) Let A, B, and C be three distinct noncollinear points. Suppose there exists two points D and E such that $A * C * D$ and $B * E * C$. Sketch a picture (or pictures).



Determine which of the following must always be true in the context of Neutral Geometry and based on the axioms, theorems, and postulates up to and including 4.3.7.

(a). D and E are on the same side of $\overleftrightarrow{AB}$. TRUE

(b). $\overline{DE}$ must intersect $\overline{AB}$. False

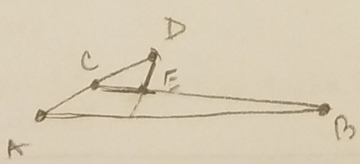
Note that it is line segment DE, not line DE.

(c). $\mu(\angle DCB) = \mu(\angle CAB) + \mu(\angle CBA)$. False

Only know
$\mu(\angle DCB) > \mu(\angle CAB)$
$\mu(\angle DCB) > \mu(\angle CBA)$

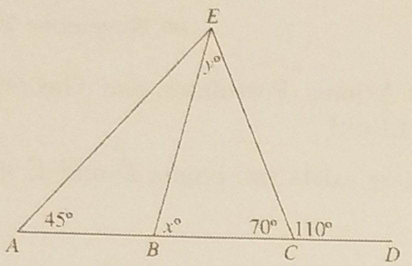
(d). $\overrightarrow{AE}$ bisects $\overline{BC}$. False

(e). $AD > BC$.



False

(f). $\angle BED$ is obtuse. False ↑ See picture



2. (12 pts) Given the figure above. find upper and lower bounds for

(a). $\mu(\angle EBC) = x$

By the Ext. Angle Theorem,

$x^\circ > 45^\circ$ and $110^\circ > x^\circ$

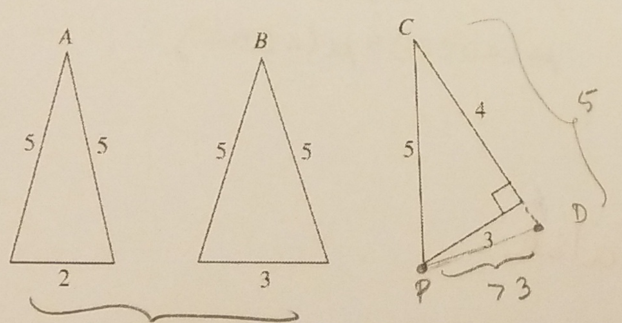
$\Rightarrow$ $\boxed{45^\circ < x^\circ < 110^\circ}$

(b). $\mu(\angle BEC) = y$.

Also by E.A.T, $110^\circ > y^\circ$. No other restrictions other than y must be an angle with positive measure

$\boxed{0^\circ < y^\circ < 110^\circ}$

3. (12 pts) Given the figure below, order the angle measures $\mu(\angle A)$, $\mu(\angle B)$ and $\mu(\angle C)$ from smallest to largest. Clearly state any theorem(s) to justify your answer.



Construct D s.t. CD = 5
Then by Theorem 4.3.4,
PD > 3

$\boxed{\text{By the Hinge Theorem } \mu(\angle A) < \mu(\angle B) < \mu(\angle C)}$

4. (12 pts) Fill in the blanks to prove the Triangle Inequality: If A, B, and C are 3 noncollinear points, then $AC < AB + BC$.

PROOF: Let A, B, and C be 3 noncollinear points and define $\triangle ABC$.

By point construction, let D be the point on $\overrightarrow{AB}$ such that $A * B * D$ and $BD = BC$.

Since B is between A and D, then $\overrightarrow{CB}$ is between $\overrightarrow{CA}$ and $\overrightarrow{CD}$ by betweenness for rays

$\Rightarrow \mu(\angle ACB) + \mu(\angle BCD) = \mu(\angle ACD)$ by angle-addition.

$\Rightarrow \mu(\angle ACD) > \mu(\angle BCD)$. (*)

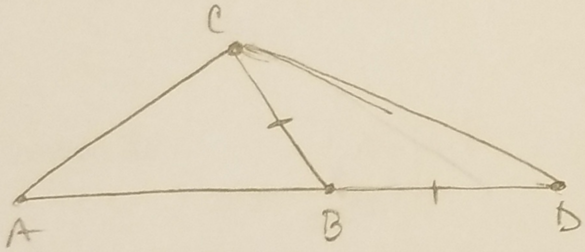
Since $\triangle BCD$ is Isosceles, then $\mu(\angle BCD) = \mu(\angle BDC)$. (**)

But $\angle BDC = \angle ADC$

Combining (*) and (**) $\Rightarrow \mu(\angle ACD) > \mu(\angle ADC)$.

Then $AB + BD > AC$ by the Scalene Inequality.

Therefore $AB + BC > AC$ since $BD = BC$. ■



5. (24 pts) Prove 2 of the following. Clearly state theorems and properties that you use.
BONUS: You may do (or attempt) all three options and each will be graded out of 12 points. Whichever two you score higher on will be your base grade. Any points from the third problem will be cut in third and added to your base grade.

(a). (Old) Let A, B, and C be three noncollinear points and let P be a point in the interior of $\angle BAC$. Also suppose that P lies on the angle bisector of $\angle BAC$. Prove that $d(P, \overleftrightarrow{AB}) = d(P, \overleftrightarrow{AC})$. [Note: This is one direction of Theorem 4.3.6.]

(b). (Old) Prove THEOREM 3.5.12 Vertical Angles Theorem (Restated): If $\angle BAC$ and $\angle DAE$ form a vertical pair, then $\angle BAC \cong \angle DAE$.

(c). (New) Let $\triangle HJK$ be an equilateral triangle. Suppose that $G * J * K * M$ such that $\overline{GJ} \cong \overline{KM}$. Prove that $\overline{HG} \cong \overline{HM}$.

6. (24 pts) The take-home portion of the exam is due Thursday, November 8, 2018 beginning of class.

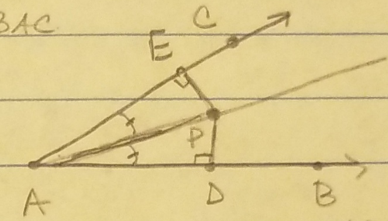
Exam 2

5(a) Let A, B, C be 3 noncollinear points and let P be a point in the interior of $\angle BAC$.

Let P lie on the angle bisector of $\angle BAC$. [Show $d(P, \overleftrightarrow{AB}) = d(\overleftrightarrow{AC})$]

ie $\overrightarrow{AP}$ is the angle bisector of $\angle BAC$

$\Rightarrow \angle BAP \cong \angle CAP$ (*)



Drop perpendiculars from P to $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$.

Let D & E be the feet, respectively. Then $d(P, \overleftrightarrow{AB}) = PD$ & $d(P, \overleftrightarrow{AC}) = PE$

Then $\mu(\angle PDA) = 90°$ & $\mu(\angle PEA) = 90°$ by def. of perpendicular

$\Rightarrow \angle PDA \cong \angle PEA$ (**)

Using (*), (**) and the common side $\overline{AP}$,

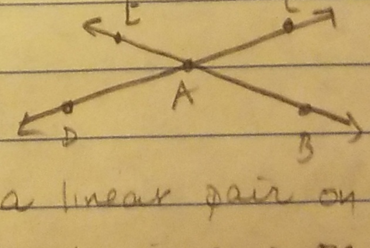
$\triangle APE \cong APD$ by AAS

$\Rightarrow \overline{PD} \cong \overline{PE} \Rightarrow PD = PE$

$\therefore d(P, \overleftrightarrow{AB}) = d(P, \overleftrightarrow{AC})$ by definition of distance from a point to a line ■

5(b) Let $\angle BAC$ & $\angle DAE$ form a vertical pair. [Show $\angle BAC \cong \angle DAE$]

WLOG, let $\overrightarrow{AB}$ + $\overrightarrow{AE}$ be opposite rays & $\overrightarrow{AC}$ & $\overrightarrow{AD}$ be opposite rays from the def. of vertical pairs.



Then $\angle BAC$ and $\angle CAE$ form a linear pair on $\overleftrightarrow{BE}$ and $\angle CAE$ and $\angle EAD$ form a linear pair on $\overleftrightarrow{CD}$

$\Rightarrow \mu(\angle BAC) + \mu(\angle CAE) = 180°$ and $\mu(\angle CAE) + \mu(\angle EAD) = 180°$

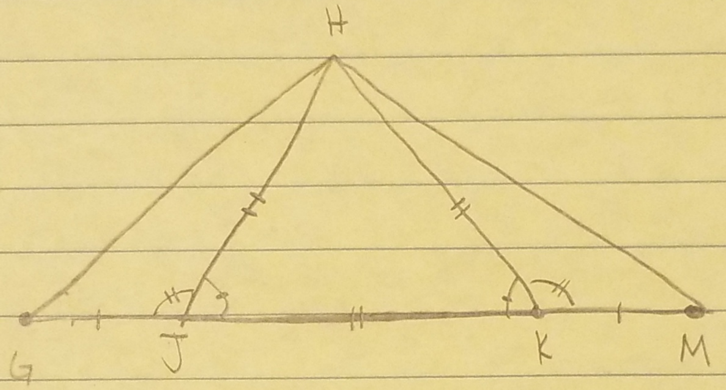
Set them equal $\Rightarrow \mu(\angle BAC) + \mu(\angle CAE) = \mu(\angle CAE) + \mu(\angle EAD)$

$\Rightarrow \mu(\angle BAC) = \mu(\angle EAD)$. But $\mu(\angle EAD) = \mu(\angle DAE)$ ← same angle

$\Rightarrow \angle BAC \cong \angle DAE$ ■

5(c) Proof Let $\triangle HJK$ be an equilateral triangle.
Let $G*J*K*M$ s.t. $\overline{GJ} \cong \overline{KM}$. [Show $\overline{HG} \cong \overline{HM}$]

Picture



(Since $\triangle HJK$ is equilateral, it is also isosceles.)

By the Isosceles Triangle Theorem $\angle HJK \cong \angle HKJ$ $\Rightarrow \mu(\angle HJK) = \mu(\angle HKJ)$ $(*)$

Then $\angle MKH$ forms a ~~vertical~~ linear pair w/ $\angle HKJ$

$\Rightarrow \mu(\angle MKH) + \mu(\angle HKJ) = 180° \Rightarrow \mu(\angle MKH) = 180° - \mu(HKJ)$ $(**)$

Also $\angle GJH$ forms a ~~vertical~~ linear pair w/ $\angle HJK$

$\Rightarrow \mu(\angle GJH) + \mu(\angle HJK) = 180°$

$\Rightarrow \mu(\angle GJH) + \mu(HKJ) = 180°$ by $(*)$

$\Rightarrow \mu(\angle GJH) = 180° - \mu(\angle HKJ)$ $(***)$

$\Rightarrow \mu(\angle GJH) = \mu(\angle MKH)$ by $(**)$ and $(***)$

$\therefore \angle GJH \cong \angle MKH$

$\Rightarrow \triangle GJH \cong \triangle MKH$ by SAS

$\therefore \overline{HG} \cong \overline{HM}$ by def. of congruent triangles. $\blacksquare$

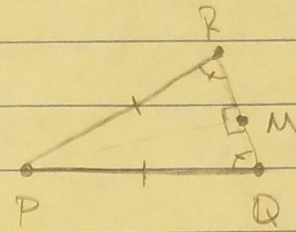
Exam 2 Take-Home

Proof

1. Let $\triangle PQR$ be a triangle s.t. $PQ = PR$.

[Show $\exists$ M s.t. $\overleftrightarrow{PM} \perp \overline{QR}$ & $\overleftrightarrow{PM}$ bisects $\overline{QR}$]

Drop a perpendicular from P to $\overline{QR}$ and call the foot M



Then $\overleftrightarrow{PM} \perp \overleftrightarrow{QR}$ by def of perpendicular.

$\Rightarrow \mu(\angle PMQ) = \mu(\angle PMR) = 90°$

Also, since $\overline{PQ} \cong \overline{PR}$, then $\angle PQM \cong \angle PRM$ by the I.T.T.

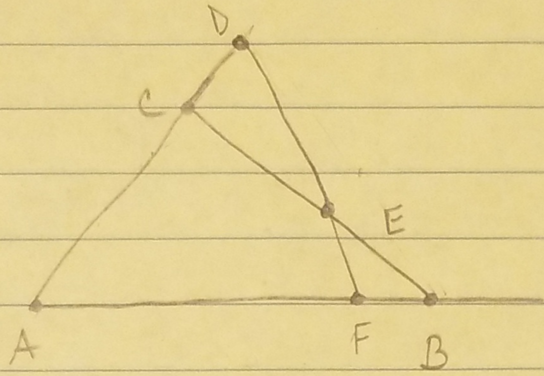
$\Rightarrow \triangle PMR \cong \triangle PMQ$ by AAS

$\Rightarrow \overline{RM} \cong \overline{QM} \Rightarrow RM = QM$

$\Rightarrow \overleftrightarrow{PM}$ bisects $\overline{QR}$ ■

2. Proof Let A, B, & C be 3 distinct noncollinear points.
Let D & E be 2 points s.t. $A*C*D$ & $B*E*C$

Picture



(a) [Show $\exists F$ on $\overline{AB}$ s.t. $A*F*B$ & $D*E*F$]

By definition of betweenness A, C, & D are collinear and distinct
Similarly B, E, & C are collinear and distinct. $\Big\}(*)$

Then $l = \overleftrightarrow{DE}$ does not go through A, B, or C otherwise some part of (*) would fail

Since l intersects $\overline{BC}$ at E it must also intersect either $\overline{AB}$ or $\overline{AC}$ by Pasch's Theorem (3.3.12)

It cannot intersect $\overline{AC}$ otherwise $B*E*C$ would not hold

$\Rightarrow$ l intersects AB at a point F, which is not A or B

$\therefore$ $A*F*B$ (from above)

Also $D*E*F$ otherwise $B*E*C$ would fail

∎

(b) [Show that F is unique]

BWOC, Suppose $\exists$ 2 pts F & F' on $\overline{AB}$ s.t. $A*F*B$ & $D*E*F$
& $A*F'*B$ & $D*E*F'$

$\Rightarrow$ $\overleftrightarrow{DE}$ intersects $\overline{AB}$ at 2 pts F & F' $\rightarrow\!\leftarrow$

$\therefore$ The point F is unique ∎