Name: ______________________________ **Exam 2**

Math 331 Foundations of Geometry – Crawford **05 November 2018**

Books and notes (in any form) are not allowed. You may use the provided list of Axioms, Postulates, and Theorems. ***Show all your work.*** Partial credit may be given for written work. GOOD LUCK!

1. (16 pts) Let A, B, and C be three distinct noncollinear points. Suppose there exists two points D and E such that $A * C * D$ and $B * E * C$. Sketch a picture (or pictures).

Determine which of the following must always be true in the context of Neutral Geometry and based on the axioms, theorems, and postulates up to and including 4.3.7.

(a). D and E are on the same side of $\overleftrightarrow{AB}$.

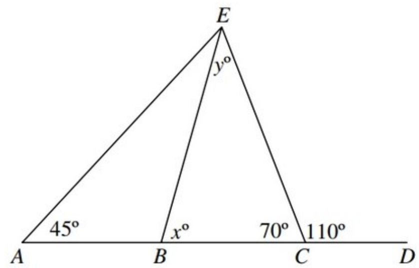
(b). $\overline{DE}$ must intersect $\overline{AB}$.

(c). $\mu(\angle DCB) = \mu(\angle CAB) + \mu(\angle CBA)$.

(d). $\overrightarrow{AE}$ bisects $\overline{BC}$.

(e). $AD > BC$.

(f). $\angle BED$ is obtuse.

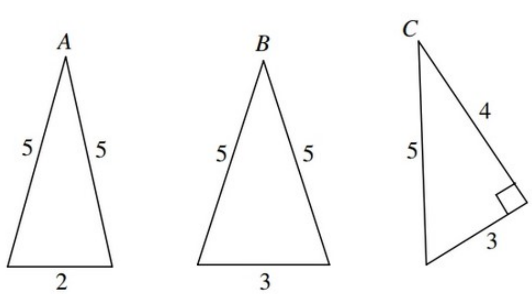


2. (12 pts) Given the figure above, find upper and lower bounds for

(a). $\mu(\angle EBC) = x$

(b). $\mu(\angle BEC) = y$.

3. (12 pts) Given the figure below, order the angle measures $\mu(\angle A), \mu(\angle B)$ and $\mu(\angle C)$ from smallest to largest. Clearly state any theorem(s) to justify your answer.



4. (12 pts) Fill in the blanks to prove the Triangle Inequality: If A, B, and C are 3 noncollinear points, then $AC < AB + BC$.

<u>Proof</u>: Let A, B, and C be 3 noncollinear points and define $\triangle ABC$.

By ________________________ , let D be the point on $\overrightarrow{AB}$ such that $A * B * D$ and $BD = BC$.

Since B is between A and D, then $\overrightarrow{CB}$ is between __________ and __________ by _____________________

$\Rightarrow \mu(\angle ACB) + \mu(\angle BCD) =$ ________________ by ________________________________ .

$\Rightarrow \mu(\angle ACD)$________$\mu(\angle BCD)$. (*)

Since $\triangle BCD$ is ______________ , then $\mu(\angle BCD) =$ ______________________. (**)

Combining (*) and (**) $\Rightarrow \mu(\angle ACD) > \mu(\angle ADC)$.

Then ______________________ by the Scalene Inequality.

Therefore $AB + BC > AC$ since ________________ . ∎

5. (24 pts) <u>Prove 2</u> of the following. Clearly state theorems and properties that you use.
<u>Bonus</u>: You may do (or attempt) all three options and each will be graded out of 12 points. Whichever two you score higher on will be your base grade. Any points from the third problem will be cut in third and added to your base grade.

(a). (Old) Let A, B, and C be three noncollinear points and let P be a point in the interior of $\angle BAC$. Also suppose that P lies on the angle bisector of $\angle BAC$. Prove that $d(P, \overleftrightarrow{AB}) = d(P, \overleftrightarrow{AC})$. [Note: This is one direction of Theorem 4.3.6.]

(b). (Old) Prove Theorem 3.5.12 Vertical Angles Theorem (Restated): If $\angle BAC$ and DAE form a vertical pair, then $\angle BAC \cong \angle DAE$.

(c). (New) Let $\triangle HJK$ be an equilateral triangle. Suppose that $G * J * K * M$ such that $\overline{GJ} \cong \overline{KM}$. Prove that $\overline{HG} \cong \overline{HM}$.

6. (24 pts) The take-home portion of the exam is due Thursday, November 8, 2018 beginning of class.