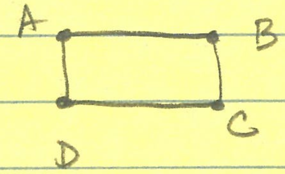
Exam 1 - Key

1. Points $\{A, B, C, D\}$ Lines: $\{A,B\}, \{B,C\}, \{C,D\}, \{A,D\}$

AC & BD don't have a line

(a)



(b)

IA 1 - No

IA 2 - Yes

IA 3 - Yes

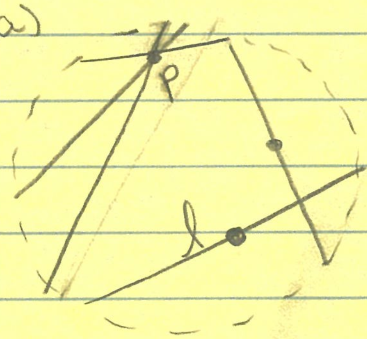
EucPP - Yes

Ell PP - No

Hyp PP - No

2. Klein disk

(a)



(b) Hyp. PP holds

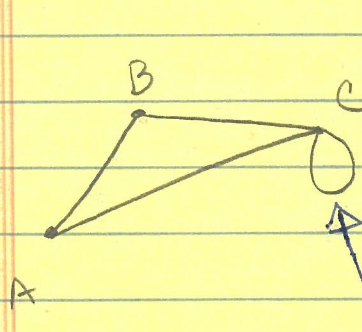
eg line l and point P:

There are infinitely many lines through P (not on l), s.t. l is parallel to those lines.

(3 parallel lines shown)

Similar for any line & any pt not on that

3.



Points: $\{A, B, C\}$

Lines: $\{A,B\}, \{A,C\}, \{B,C\}, \{C\}$

(Other similar type schematics are possible)

4\. - See cover sheet.

5\. (a) If we do not eat turkey and do not eat tofurkey then it is not Thanksgiving.

(b) For all lines m, m is not parallel to ℓ.

(c) If $a=0$ or $b=0$, then $ab=0$.

6\. $H \Rightarrow C$ $\quad$ (not H) or C

(a)

H	C	not H	$H \Rightarrow C$	(not H) or C
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

(b) $\boxed{\text{Yes}}$, the are logically equiv.

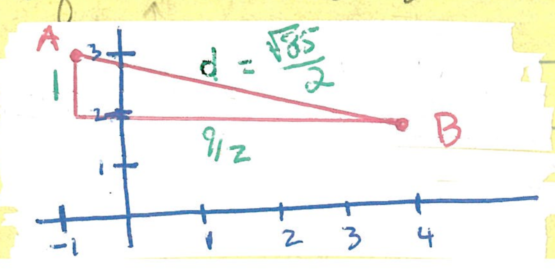
7\. $A(-\tfrac{1}{2}, 3)$ $\quad B(4,2)$

Euclidean metric: $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(4+\frac{1}{2})^2 + (2-3)^2} = \sqrt{(\frac{9}{2})^2 + (-1)^2} = \sqrt{\frac{81}{4}+1} = \sqrt{\frac{85}{4}} = \boxed{\frac{\sqrt{85}}{2}}$

taxicab metric: $f = |x_2-x_1| + |y_2-y_1| = |\frac{9}{2}| + |-1| = \frac{9}{2}+1 = \boxed{\frac{11}{2}}$

square metric: $D = \max\{|x_2-x_1|, |y_2-y_1|\} = \max\{\frac{9}{2}, 1\} = \boxed{\frac{9}{2}}$



8. Let $p, q, \& r$ be the coordinates in $\mathbb{R}$ of $P, Q, \& R$ respectively

System 1: $PQ = |p-q| = |-5-7| = 12$
System 2: $PR = |p-r| = |5-(-3)| = 8$
System 3: $QR = |q-r| = |-8-(-12)| = 4$

Since $P, Q, \& R$ are collinear and $PR + QR = PQ$
$\Rightarrow PR + RQ = PQ$
$8 + 4 = 12$,
then by definition of between, $\boxed{R \text{ is between } P \& Q}$

(see next page for 9(a-b))

9(c) If $P \& Q$ are two points s.t $P \neq Q$, then there exists a point R s.t. $P, Q, \& R$ are noncollinear.
<u>Proof</u>. Let $P \& Q$ be 2 points s.t $P \neq Q$.
By IA1, there exists a line l s.t. $P \& Q$ lie on l.
By Theorem 2.6.2, there exists a point R s.t. R does not lie on l.
$\therefore$ $P, Q, \& R$ are noncollinear.

9(a) If $A*B*C$, $D*E*F$, $\overline{AB} \cong \overline{DE}$, & $\overline{BC} \cong \overline{EF}$, then $\overline{AC} \cong DF$

Proof Let $A*B*C$, $D*E*F$, $\overline{AB} \cong \overline{DE}$, & $\overline{BC} \cong EF$

Then $A, B, \& C$ are collinear & $AB + BC = AC$ (*)

Also $D, E, \& F$ " " " $DE + EF = DF$ (**)

Since $\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong \overline{EF}$ $\Rightarrow$ $AB = DE$ & $BC = EF$

by def. of congruence.

Then $\overset{(*)}{} AC = AB + BC$

$= DE + EF$

$= DF$ by (**)

$\therefore$ $\overline{AC} \cong \overline{DF}$ by definition of congruence. ∎

(b) Let A & B be two distinct points. Prove equality of sets $\overline{AB} = \overline{BA}$.

Proof Let A, B be 2 distinct points

Let $P \in \overline{AB}$. [Show $P \in \overline{BA}$]

Then $P = A$ or $P = B$ or $A*P*B$ by def of $\overline{AB}$

$\Rightarrow$ $P = A$ or $P = B$ or $B*P*A$ by Corollary 3.2.8

$=$ $P \in \overline{BA}$ by def. of $\overline{BA}$.

$\Rightarrow$ $\overline{AB} \subseteq \overline{BA}$ (*)

Let $P \in \overline{BA}$ [Show $P \in \overline{AB}$]

Then $P = B$ or $P = A$ or $B*P*A$ by def

$\Rightarrow$ $P = B$ or $P = A$ or $A*P*B$ by Cor. 3.2.8

$\Rightarrow$ $P \in \overline{AB}$ by def

$\Rightarrow$ $\overline{BA} \subseteq \overline{AB}$ (*)

Then by (*) + (**) $\overline{AB} = \overline{BA}$ ∎

Easier Proof: Let A, B be 2 distinct points.

Then $\overline{AB} = \{A, B\} \cup \{P \mid A*P*B\}$ by def

$= \{A, B\} \cup \{P \mid B*P*A\}$ by Cor. 3.2.8

$= \overline{BA}$ by def. ∎