Name: ________________________________ **Exam 1**

Math 331 Foundations of Geometry – Crawford **27 September 2018**

Books and notes (in any form) are not allowed. You may use the provided list of Axioms, Postulates, and Theorems. ***Except for problem #4, put all of your work and answers on other sheets of paper.*** Include this sheet as a cover sheet. ***Show all your work***. Partial credit may be given for written work. GOOD LUCK!

1. (12 pts) Given the following model, Points: $\{A, B, C, D\}$ Lines: $\{A, B\}, \{B, C\}, \{C, D\}, \{A, D\}$

(a). Sketch a schematic diagram of this model.

(b). Determine which of the Incidence Axioms hold and which of the Parallel Postulates hold.

2. (10 pts) Recall the Klein disk: Interpret *point* to mean a point in the Cartesian plane that lies inside the unit circle (i.e., (x, y) such that $x^2 + y^2 < 1$). A *line* is the part of a Euclidean line that lies inside the unit circle. *Lie on* has its usual Euclidean meaning (a point lies on the Euclidean line).

(a). Sketch the Klein disk. Include a few examples of points and lines.

(b). Determine which of the Parallel Postulates hold.

3. (6 pts) Find interpretations for the words *point*, *line*, and *lie on* such that Incidence Axioms 1 and 3 hold, but Axiom 2 does not. If you want, you may answer this question by drawing a correct schematic diagram that illustrates the required interpretations.

4. (8 pts) Determine whether the following statements are true for

- **ALL** models of incidence geometry.
- **SOME** models of incidence geometry, but not all.
- **NONE** of the models of incidence geometry. (i.e., False for all models of incidence geometry.)

[Circle the correct answer below.]

ALL SOME NONE If P and Q are distinct points that lie on a line l, then there exists another point R, distinct from P and Q, that also lies on l.

ALL SOME NONE Every point lies on at least two distinct lines.

ALL SOME NONE There exists a line that contains exactly one point.

ALL SOME NONE There exists a line that contains exactly two points.

5. (12 pts)

(a). Write the contrapositive of the statement: If it is Thanksgiving, then we eat turkey or tofurkey.

(b). Write the negation of the statement: There is a line m such that m is parallel to l.

(c). Write the converse of the statement: If $ab = 0$, then $a = 0$ or $b = 0$.

6. (12 pts) Given the two statements $\underline{H \Rightarrow C}$ and $\underline{(\text{not } H) \text{ or } C}$

(a). Construct truth table(s) for these statements.

(b). Are the statements logically equivalent?

7. (12 pts) Given the points $A(-1/2, 3)$ and $B(4, 2)$ in the Cartesian plane, find the distance between A and B using the Euclidean metric d, the taxicab metric ρ, and the square metric D. If any of these distances are undefined, clearly explain why.

8. (10 pts) Three distinct points P, Q, and R all lie on the same line. Three different coordinate systems are set up for the line such that the following hold:

- In the first system, the coordinate of P is -5 and the coordinate of Q is 7.
- In the second system, the coordinate of P is 5 and the coordinate of R is -3.
- In the third system, the coordinate of Q is -8 and the coordinate of R is -12.

(a). Which point is between the other two? [Justify your answer.]

(b). Evaluate $PQ + QR + RP$.

9. (20 pts) Prove 2 of the following. Clearly state theorems and properties that you use.
BONUS: You may do (or attempt) all three options and each will be graded out of 10 points. Whichever two you score higher on will be your base grade. Any points from the third problem will be cut in third and added to your base grade.

(a). (Old) If $A * B * C$, $D * E * F$, $\overline{AB} \cong \overline{DE}$, and $\overline{BC} \cong \overline{EF}$, then $\overline{AC} \cong \overline{DF}$.

(b). (New) Let A and B be two distinct points. Prove the following equality of sets. $\overline{AB} = \overline{BA}$

(c). (Newish) If P and Q are two points such that $P \neq Q$, then there exists a point R such that P, Q, and R are noncollinear. This is Theorem 2.6.9. Prove it using any theorems, postulates, and/or axioms that come before it.