DEF Three lines (or segments) are said to be CONCURRENT if there is a point P that lies on all three lines (or segments). The point P is the POINT OF CONCURRENCY.

[Sketch]

1. Use Geometer's Sketchpad to draw a triangle $\triangle ABC$.

(a). Construct and label the midpoints of each side.

(b). Connect each vertex of the triangle to the midpoint of the opposite side.

(c). What property do you notice?
If you move the vertices of the triangle around, does the property still hold?

(d). Label this intersection point G.

(e). Measure the distance from each vertex to the intersection point, then from the intersection point to the midpoint on the opposite side.

(f). What property do you notice?
If you move the vertices of the triangle around, does the property still hold?

DEF A MEDIAN for a triangle is a segment joining a vertex of the triangle to the midpoint of the opposite side.

[Fill in the blanks for the following theorem.]

THEOREM (Median Concurrence Theorem). The three medians of any triangle are ___concurrent___ . The point of concurrency is called the CENTROID of the triangle. Furthermore, the centroid divides the medians in a ratio of ___2:1___ with the longer segment near the vertex and the shorter segment near the midpoint.

[In GSP, label the centroid G and hide the midpoints and medians.]

2. DEF An altitude for a triangle is a line through one vertex that is perpendicular to the line determined by the opposite two vertices.

(a). Using the already constructed triangle, construct the three altitudes. What property do you notice?

(b). Does the property hold or change for acute or obtuse triangles?

[Fill in the blanks for the following theorem.]

THEOREM (Altitude Concurrence Theorem). The three altitudes of any triangle are ___concurrent___ . The point of concurrency is called the ORTHOCENTER of the triangle.

[In GSP, label the orthocenter H and hide the altitudes.]

3. (a). Using the already constructed triangle, construct the three perpendicular bisectors. What properties do you notice? Are there any properties about the distances?

(b). Do these property hold or change for acute or obtuse triangles?

[Fill in the blanks for the following theorem.]

THEOREM (Perpendicular Concurrence Theorem). The three perpendicular bisectors of any triangle are ___concurrent___ . The point of concurrency is called the CIRCUMCENTER of the triangle. Furthermore, the distances from the circumcenter to each vertex are ___equal___ .

[In GSP, label the circumcenter O and hide the perpendicular bisectors.]

4. Move the vertices of the triangle around.

(a). What property do you notice about the three centers G, H, and O?

(b). Measure the distances HG and GO. Then calculate HG/GO. Move the vertices around to see what property holds.

[Fill in the blanks for the following theorem.]

THEOREM (Euler Line Theorem). The centroid G, the circumcenter H, and the orthocenter O of any triangle are ___collinear___ . This common line is called the EULER LINE. Furthermore, if the triangle is not equilateral, G is between H and O and $HG =$ ___$2\,GO$___.

What happens if the triangle is equilateral?