State and prove the following corollary to the Converse to the Alternate Interior Angles Theorem:

<u>Theorem</u> (Converse to the Corresponding Angles Theorem)

<u>Theorem</u> (Parallel Projection Theorem) Let l, m, and n be distinct parallel lines. Let t be a transversal that cuts these lines at A, B, and C, respectively, and assume $A * B * C$. Let t' be a transversal that cuts these lines at A', B', and C', respectively. Then $\frac{AB}{AC} = \frac{A'B'}{A'C'}$.

[Sketch]

<u>Proof</u> (Given w/o proof – you can look it up in Section 5.2 later).

<u>Def</u> Triangles $\triangle ABC$ and $\triangle DEF$are ___similar___ if $\angle ABC \cong \angle DEF, \angle BCA \cong \angle EFD$, and $\angle CAB \cong \angle FDE$.

Denoted $\triangle ABC \sim \triangle DEF$.

Sketch two similar triangles that are not congruent.

<u>THEOREM</u> (Fundamental Theorem of Similar Triangles) If $\triangle ABC \sim \triangle DEF$, then $\dfrac{AB}{DE} = \dfrac{AC}{DF} = \dfrac{BC}{EF}$.

<u>PROOF</u> Let $\triangle ABC \sim \triangle DEF$. [Show the first equality $\dfrac{AB}{DE} = \dfrac{AC}{DF}$]

<u>Case 1</u> $AB = DE$. Then $\triangle ABC \cong \triangle DEF$ by ___ASA___ .

Then $AC = DF$ and therefore, $\dfrac{AB}{DE} = \dfrac{AC}{DF} =$ ___1___.

<u>Case 2</u> $AB \neq DE$.

WLOG, let $AB > DE$. [Sketch. Include congruent angles.]

Then by the Ruler Postulate, construct a point B' on AB such that ___$AB' = DE$___ .

Then B' is external to $l = \overleftrightarrow{BC}$ and by the EPP, there exists a unique line m through ___B'___ such that ___$m \parallel l$___ .

By Pasch's Axiom, m must intersect ___$\overline{AC}$___ (since m is parallel to ___$\overleftrightarrow{BC}$___). Let C' be this point of intersection.

Then by the ___Converse to the Corresponding Angles Theorem___ , $\angle AB'C' \cong \angle ABC$.

Therefore $\triangle AB'C' \cong$ ___$\triangle DEF$___ by ___ASA___ .

By the EPP, construct the unique line n through A such that n is parallel to ___m and l___ .

By the Parallel Projection Theorem, $\dfrac{AB'}{AB} =$ ___$\dfrac{AC'}{AC}$___ .

$\Rightarrow \dfrac{DE}{AB} = \dfrac{DF}{AC}$ by congruent triangles.

Or equivalently, $\dfrac{AB}{DE} = \dfrac{AC}{DF}$.

The proof of the second equality is similar. ■

State and Prove:

<u>Theorem</u> (Converse to the Similar Triangles Theorem)

[Hint: Use cases like previous theorem. Use the Converse to Corresponding Angles Theorem and SSS.]

<u>Def</u> If $\triangle ABC \sim \triangle DEF$, then the number $r = \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$, is called the <u>common ratio</u> of the sides of the similar triangles.

<u>Theorem</u> (SAS Similarity Criterion) If $\triangle ABC$ and $\triangle DEF$ are two triangles such that $\angle CAB \cong \angle FDE$ and $\frac{AB}{AC} = \frac{DE}{DF}$, then $\triangle ABC \sim \triangle DEF$.

<u>Proof</u> Given w/o proof (but proof has similarities to Similar Triangles Theorem and its Converse.)

<u>THEOREM</u> (ASA Similarity Criterion) If $\triangle ABC$ and $\triangle DEF$ are two triangles such that $\angle CAB \cong \angle FDE, \angle ABC \cong \angle DEF$ and $DE = r \cdot AB$, then $\triangle ABC \sim \triangle DEF$ with common ratio r.

<u>PROOF</u>

Standard Notation for triangles: Let a, b, and c denote the length of the sides opposite vertices A, B, and C, respectively.

<u>THEOREM</u> (The Pythagorean Theorem) If $\triangle ABC$ is a right triangle with right angle at C, then $a^2 + b^2 = c^2$.

[Hint for proof: Drop a perpendicular from C and show that you have 3 similar triangles. May need Lemma 4.8.6 (w/o proof).]

<u>PROOF</u>

Homework: Finish worksheet from 11/24 and today's worksheet. Skim Sections 5.1-5.3 to note additional theorems we haven't proved.