<u>THEOREM</u> (Alternate Interior Angles Theorem): If l and m are two lines cut by a transversal t in such a way that a pair of alternate interior angles is congruent, then $l \parallel m$.

Sketch a diagram.

<u>PROOF</u> Let l and m be two lines cut by a transversal t in such a way that a pair of alternate interior angles is congruent. [Show that $l \parallel m$.]

Choose points A, B, C and A', B', C' as in the definitions of transversal and interior angles.

WLOG, let the congruent alternate interior angles be $\angle A'B'B$ and $\angle B'BC$. i.e. $\angle A'B'B \cong \angle B'BC$

BWOC, suppose l and m ____________________ .

Then l and m must ______________ . Let D be this point of ________________ .

<u>Case 1</u>: D lies on the same side of t as C. [Sketch a new picture.]

Then $\angle A'B'B$ is an __________________ for $\triangle BB'D$.

Then $\mu(\angle A'B'B) > \mu(\angle B'BD)$ by the ____________________________ .

But $\angle B'BD =$ _____________ $\Rightarrow \mu(\angle A'B'B) > \mu($_____________$)$ $\rightarrow\leftarrow$ since _______________________ .

<u>Case 2</u>: D lies on the same side of t as A.

Then by a similar argument $\mu(\angle B'BC) > \mu(\angle DB'B) =$ ______________ $\rightarrow\leftarrow$.

Both cases lead to a contradictions. Therefore, ___________ . ■

<u>COROLLARY</u> (Corresponding Angles Theorem): If l and m are two lines cut by a transversal t in such a way that a pair of corresponding angles is congruent, then $l \parallel m$.

Sketch a diagram.

Hint for proof: You could prove this in a manner similar to proof of the above theorem, but it's called a corollary for a reason: use the previous theorem to prove it.

<u>PROOF</u>

<u>COROLLARY</u> (Existence of Parallels Theorem): If l is a line and P a point with $P \notin l$, then there exists a line m with $P \in m$ and $l \parallel m$.

Sketch a diagram.

Hint for proof: Drop a perpendicular from P to l and remember you are proving a corollary to the Alternate Interior Angles Theorem.

<u>PROOF</u>

<u>Lemma 4.5.3</u> (Two-Angles Sum): If $\triangle ABC$ is any triangle, then $\mu(\angle CAB) + \mu(\angle ABC) < 180°$.

Sketch a diagram.

Hint for proof: This follows almost immediately by the Exterior Angle Theorem.

<u>Proof</u>

<u>Lemma 4.5.4</u> (Adjacent Triangles Angle Sum): If $\triangle ABC$ is any triangle and E is distinct from B and C with $E \in \overline{BC}$, then $\sigma(\triangle ABE) + \sigma(\triangle ECA) = \sigma(\triangle ABC) + 180°$.

Sketch a diagram.

<u>Proof</u>

<u>LEMMA 4.5.5</u> (Equal Angle Sum): If $\triangle ABC$ is any triangle then there exists a point $D \notin \overleftrightarrow{AB}$ such that $\sigma(\triangle ABD) = \sigma(\triangle ABC)$ and one of the interior angles of $\triangle ABD$ is less than or equal to $\frac{1}{2}\mu(\angle CAB)$.

Sketch a diagram.

Hint for proof: Let M be the midpoint of $\overline{BC}$ and let $A * M * D$ with $AM = MD$.

<u>PROOF</u>

These lemmas will all be used to prove the following theorem. Close your book and fill in the blank below. Then look up the statement of the theorem on p. 85.

<u>THEOREM</u> (Saccheri-Legendre Theorem): If $\triangle ABC$ is any triangle, then $\sigma(\triangle ABC)$ ________ $180°$.

Is the conclusion of this theorem what you expected?

If not, why do you think it is different?

<u>PROOF</u> Done together as a class.

Homework: Finish the proofs on this worksheet. Read the text of Sections 4.4-4.5 and summarize the main points.