Some theorems we already have:

<u>Theorem 3.4.7 (Existence and Uniqueness of Angle Bisectors)</u> If A, B, and C are three noncollinear points, then there exists a unique angle bisector for $\angle BAC$.

<u>Theorem 3.5.9</u> If l is a line and P is a point on l, then there exists exactly one line m such that P lies on m and $m \perp l$.

<u>Theorem 3.5.11 (Existence and Uniqueness of Perpendicular Bisectors)</u> If D and E are two distinct points, then there exists a unique perpendicular bisector for $\overline{DE}$.

Some new theorems:

<u>Theorem 4.1.3 (Existence and Uniqueness of Perpendiculars)</u> For every line l and for every point P, there exists a unique line m such that P lies on m and $m \perp l$.

Sketch picture(s) and explain the difference or similarity between Theorems 3.5.9 and 4.1.3.

<u>Terminology</u> : By 4.1.3 we can say, "drop a perpendicular from P to l." Also the point F that where the perpendicular intersects l is called the <u>foot</u> (of the perpendicular).

[Case 2 picture below.]

<u>Proof</u> Let l be a line and P be a point.

<u>Case 1</u>: P is on l. Then the conclusion is true by Theorem ___3.5.9___ .

<u>Case 2</u>: P is not on l. [Sketch a picture.] [Existence: Show m exists s.t. ___$P \in m$___ and ___$m \perp l$___ .]

Let Q and Q' be two distinct points on l and define the angle $\angle Q'QP$.

By Angle Construction, there exists a point R on the opposite side of l from P such that ___$\angle Q'QP$___ $\cong \angle Q'QR$.

By Point Construction, let P' be a point on $\overrightarrow{QR}$ such that $\overline{QP} \cong \overline{QP'}$.

Let $m =$ ___$\overleftrightarrow{PP'}$___. [By construction, $P \in m$, but we still need to show $m \perp l$.]

By Plane Separation, $l \cap \overline{PP'}$ ___$\neq \emptyset$___. Let F be this point of intersection.

Continued $\longrightarrow$

Subcase (a): $F = Q$. [Resketch (include R)]

Then $\angle Q'FP = \angle Q'QP$ and $\angle Q'FP' = \angle Q'QP'$ form a linear pair.

Thus $\mu(\angle Q'QP) + \mu(\angle Q'QP') = 180$.

But since $\angle Q'QP \cong \angle Q'QR$ and $\angle Q'QR = \angle Q'QP'$, then $\angle Q'QP \cong$ $\angle Q'QP'$.

Two congruent angles that sum to 180 must each equal 90. Therefore $m \perp l$.

Subcase (b): $F \neq Q$ and F lies on the ray $\overrightarrow{QQ'}$. [Resketch.]

Then $\angle PQF$ $= \angle PQQ'$ and $\angle P'QF =$ $\angle P'QQ'$.

Thus $\angle PQF \cong \angle P'QF$ since $\angle PQQ' \cong$ $\angle RQQ'$ $= \angle P'QQ'$.

Therefore, $\triangle FQP \cong$ $\triangle FQP'$ by SAS.

Thus $\angle QFP \cong$ $\angle QFP'$ by triangle congruency.

But $\angle QFP$ and $\angle QFP'$ also form a linear pair.

Congruent angles that form a linear pair, must be right angles (by the same argument as in subcase (a).

Therefore $m \perp l$.

Subcase (c): $F \neq Q$ and F lies on the ray opposite $\overrightarrow{QQ'}$. [Resketch.]

Then $\angle PQF$ and $\angle PQQ'$ form a linear pair and are thus supplements.

Similarly $\angle P'QF$ and $\angle P'QQ'$ form a linear pair and are supplements.

Therefore, since $\angle Q'QP \cong \angle Q'QP'$, then $\angle PQF \cong \angle P'QF$.

Thus, $\triangle FQP \cong \triangle FQP'$ by SAS.

The rest of the proof is the same as subcase (b). Uniqueness: [Homework.]

<u>THEOREM 4.3.4</u> Let l be a line, let P be an external point, and let F be the foot of the perpendicular from P to l. If R is any point on l that is different from F, then PR ___>___ PF.

Sketch a picture.

Restate (informal): The ___shortest___ distance from a point to a line is measured along the perpendicular.

<u>PROOF</u> Homework [Exercise 4.3.7] [Go on and come back to it, if time.]

<u>DEF</u> If l is a line and P is a point, the distance from P to l, denoted $d(P, l)$ is defined to be the distance from P to the foot of the perpendicular from P to l.

<u>THEOREM 4.3.6 (POINTWISE CHARACTERIZATION OF ANGLE BISECTOR)</u> Let A, B, and C be three noncollinear points and let P be a point in the interior of $\angle BAC$. Then P lies on the angle bisector of $\angle BAC$ iff $d(P, \overleftrightarrow{AB})$ ___=___ $d(P, \overleftrightarrow{AC})$.

Sketch a picture (one with P on the angle bisector and one with P not on it).

<u>PROOF</u> Homework [Exercise 4.3.8] [Go on and come back to it, if time.]

<u>THEOREM 4.3.7 (POINTWISE CHARACTERIZATION OF PERPENDICULAR BISECTOR)</u> Let A and B be distinct points. A point P lies on the perpendicular bisector of $\overline{AB}$ iff $PA = PB$.

Sketch a picture (one with P on the perpendicular bisector and one with P not on it).

<u>PROOF</u> Not assigned.

Summary of Homwork: Finish Uniqueness part of the proof on p.2; Section 4.3, p. 81: #(2, 3, 6, 5), 7, 8