[Close your books.]

**1.** THEOREM (The $Z$-Theorem): Let $l$ be a line and let $A$ and $D$ be distinct points on $l$. If $B$ and $E$ are points on opposite sides of $l$, then $\overrightarrow{AB} \cap \overrightarrow{DE} =$ ___$\emptyset$___.

Sketch a diagram for this theorem and fill in the blank above. [Can you see why it is called the $Z$-Theorem?]

PROOF Let $l$ be a line and let $A$ and $D$ be distinct points on $l$.

Also let $B$ and $E$ be on opposite sides of $l$.

By the (contrapositive of) the Ray Theorem, all points on $\overrightarrow{AB}$, except ___$A$___ lie in ___one half-plane___ determined by $l$,

Similarly, all points on $\overrightarrow{DE}$, except ___$D$___ lie in ___the other half-plane___ determined by $l$.

The half-planes do not intersect by the ___Plane Separation___ Postulate.

Thus the only place the rays could intersect would be at ___the endpoints___ .

But since $A$ and $D$ are ___distinct___ , $\overrightarrow{AB} \cap \overrightarrow{DE} = \emptyset$.

**2.** THEOREM (The Crossbar Theorem): Let $\triangle ABC$ be a triangle. If a point $D$ is in the interior of $\angle BAC$, then $\overrightarrow{AD} \cap \overline{BC}$ ___$\neq$___ $\emptyset$.

Sketch a diagram for this theorem and fill in the blank above. [Can you see why it is called the Crossbar Theorem?]

Fill in the blanks to (informally) restate the Crossbar Theorem: If a ray is in the ___interior___ of one of the angles of a triangle, then the ray must intersect the ___opposite___ side of the triangle.

PROOF We'll do it later as a class.

**3.** <u>THEOREM</u> A point $D$ is in the interior of $\angle BAC$ if and only if $\overrightarrow{AD} \cap \overline{BC}$ ___$\neq$___ $\emptyset$.

Sketch a diagram for this theorem and fill in the blank above.

<u>PROOF</u>

$\Rightarrow$: Let $D$ be a point in the interior of $\angle BAC$. Then $\overrightarrow{AD}$ intersects $\overline{BC}$ by the ___Crossbar Theorem___ .

$\Leftarrow$: Let $\overrightarrow{AD} \cap \overline{BC} \neq \emptyset$.

Then let $E \in \overrightarrow{AD} \cap \overline{BC}$.

Then $B * E * C$.

Thus, by Theorem 3.3.10, ___$AB$___ $*$ ___$AE$___ $*$ ___$AC$___.

Thus, $E$ is in the interior of $\angle$___$BAC$___

Since $D \in \overrightarrow{AE}$, $D$ is in the interior of $\angle BAC$ by the Ray Theorem.

**4.** <u>LEMMA</u> If $C * A * B$ and $D$ is in the interior of $\angle BAE$ then $E$ is in the interior of $\angle DAC$.

Sketch a diagram for this lemma.

<u>PROOF</u> Let $C * A * B$ and let $D$ be in the interior of $\angle BAE$.

Since $D$ is in the interior of $\angle BAE$, $D$ and $E$ are on the same side of ___$\overleftrightarrow{AB}$___ .

But since $\overleftrightarrow{AB} =$ ___$\overleftrightarrow{AC}$___, $D$ and $E$ are on the same side of ___$\overleftrightarrow{AC}$___.

By the Crossbar Theorem, ___$\overrightarrow{AD} \cap \overline{BE} \emptyset$___ .

Therefore $E$ and $B$ are on opposite sides of ___$\overleftrightarrow{AD}$___ .

Since $C * A * B$, $C$ and $B$ are on ___opposite sides___ of $\overleftrightarrow{AD}$.

Thus $C$ and $E$ are on the ___same side___ of $AD$ by the Plane Separation Postulate.

Therefore $E$ is in the interior of $\angle DAC$.

**5.** <u>THEOREM</u> (The Linear Pair Theorem): If angles $\angle BAD$ and $\angle DAC$ form a linear pair, then $\angle BAD + \angle DAC =$ ___$180^\circ$___.

Sketch a diagram for this theorem and fill in the blank above.

<u>PROOF</u> We'll do it later as a class.