THEOREM * If P is any point, then there are at least two distinct lines l and m such that P lies on both l and m.

PROOF Let __P be a point.__ [Show that there are at least two distinct lines s.t. P lies on both.]

By __IA3__ , there exists __3 noncollinear points__ , A, B, and C.

Case 1: Suppose P is one of these 3 points.

WLOG†, let $P = A$

Then P and B are __distinct points__ , and by __IA1__ , there exists a line l such that P and B lie on l.

Similarly __P and C__ are distinct points and there exists a line m such that __P and C__ lie on m

Therefore P lies on both l and m. [Still need to show that __$l \neq m$__ .]

But $l \neq m$, otherwise $P = A, B$, and C would be __collinear__ .

Therefore, there exists at least 2 distinct lines l and m such that P lies on both l and m.

Case 2: Suppose P is __not one of these 3 points__ .

The $P\&A$, $P\&B$, and $P\&C$ are are three pairs of __distinct points__ .

So by __IA1__ , there exist lines $l = \overleftrightarrow{PA}$, $m = \overleftrightarrow{PB}$, and $n = \overleftrightarrow{PC}$.

Hence P is on all three lines. [Still need to show that they are distinct.]

These lines are __all distinct__ , otherwise A, B, and C would be collinear.

Therefore, there exist at least 2 distinct lines such that P lies on both lines.

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Continue working on the homework Section 2.6, p. 34: #4, 5, 6, 7(newly added)

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*2.6.4 (Exercise #3) in the book.

† "Without Loss of Generality": You can use this method when the proof doesn't rely on the specifics of the different cases. For example, suppose $P = B$ instead. We could just relabel the points so that $P = A$. If you are uncertain whether to use WLOG, another method would be to do the proof for $P = A$ and then say a similar proof would hold for the other (sub)cases of $P = B$ and $P = C$.