

So far, we have

### INCIDENCE GEOMETRY AXIOMS

- **Incidence Axiom 1.** For every pair of distinct points  $P$  and  $Q$  there exists exactly one line  $l$  such that both  $P$  and  $Q$  lie on  $l$ .
- **Incidence Axiom 2.** For every line  $l$  there exist at least two distinct points  $P$  and  $Q$  such that both  $P$  and  $Q$  lie on  $l$ .
- **Incidence Axiom 3.** There exist three points that do not all lie on any one line.

### PARALLEL POSTULATES

[But we don't need them for what we are doing today, so I haven't restated them.]

### DEFINITIONS

- DEF Three points  $A$ ,  $B$ , and  $C$  are *collinear* if there exists one line  $l$  such that all three points  $A$ ,  $B$ , and  $C$  all lie on  $l$ . the points are *noncollinear* if there is no such line  $l$ .
- DEF Two lines  $l$  and  $m$  are said to be *parallel* if there is no point  $P$  such that  $P$  lies on both  $l$  and  $m$ .
- (Newly added) DEF Two lines are said to *intersect* if there exists a point that lies on both lines.

Prove the following theorems in Incidence Geometry using the above axioms and definitions.

1. THEOREM Lines that are not parallel intersect at one point.

Restate the theorem in the "If-then" form:

**2.** State the converse to the previous theorem and prove it.

**3.** Since both the original theorem and it's converse is true, how could the theorems be restated?