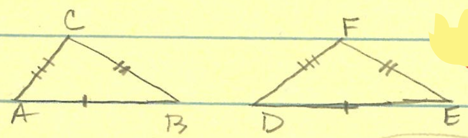
5a. Use the Hinge Theorem to prove (SSS):
If $\triangle ABC$ & $\triangle DEF$ are two triangles s.t. $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{CA} \cong \overline{FD}$, then $\triangle ABC \cong \triangle DEF$.

Proof. Let $\triangle ABC$ & $\triangle DEF$ be two triangles s.t. $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{CA} \cong FD$. [Show $\triangle ABC \cong DEF$]



[Show one angle congruency, say $\angle BAC \cong \angle EDF$, then use SAS. ∴ Show $\mu(\angle BAC) = \mu(\angle EDF)$]

(or use trichotomy)

BWOC, suppose $\mu(\angle BAC) \neq \mu(\angle EDF)$

Case 1 $\mu(\angle BAC) > \mu(\angle EDF)$

Then $BC > EF$ by the Hinge Theorem

$\times$ Since $BC = EF$ (given)

Case 2 $\mu(\angle BAC) < \mu(\angle EDF)$

Then $BC < EF$ by the Hinge Theorem

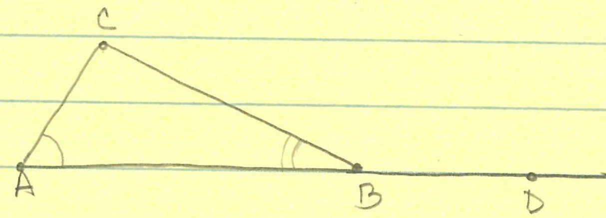
$\times$ Since $BC = EF$ (given)

Either case leads to a contradiction.

∴ $\mu(\angle BAC) = \mu(\angle EDF)$

∴ $\triangle ABC \cong \triangle DEF$ by SAS. ∎

5b Prove: If $\triangle ABC$ is any triangle, then $\mu(\angle CAB) + \mu(\angle ABC) < 180°$.



Proof. Let $\triangle ABC$ be a triangle

Construct a point D on $\overleftrightarrow{AB}$ s.t. $A * B * D$

Then $\mu(\angle ABC) + \mu(\angle CBD) = 180°$ since they form a linear pair. (*)

Also $\mu(CBD) > \mu(\angle CAB)$ by the Exterior Angle Theorem.

ie

$$\mu(\angle CAB) < \mu(\angle CBD)$$

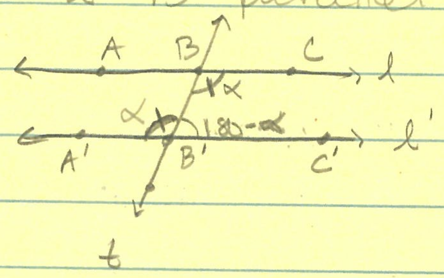
$$\Rightarrow \mu(\angle CAB) + \mu(\angle ABC) < \mu(\angle CBD) + \mu(\angle ABC) = 180°$$

by (*)

$$\Rightarrow \mu(\angle CAB) + \mu(\angle ABC) < 180° \quad \blacksquare$$

5(c) Corollary to the Alternate Interior Angles Theorem:

If l and l' are lines cut by a transversal t in such a way that two nonalternating interior angles on the same side of t are supplements, then l is parallel to l'.



Proof. Let l, l', & t be defined as above.

Let A, B, C be pts on l as shown in the figure

& A', B', C' " " " l' " " " " "

WLOG, Let $\angle B'BC$ & $\angle BB'C'$ be nonalternating interior angles on the same side of t that are supplements.

$\Rightarrow \quad \mu(\angle B'BC) + \mu(\angle BB'C') = 180 \quad (*)$

Also $\mu(\angle A'B'B) + \mu(\angle BB'C') = 180 \quad (**)$ since they form a linear pair.

Subtract these 2 equations $\Rightarrow$

$\mu(\angle B'BC) - \mu(\angle A'B'B) = 0$

$\Rightarrow \mu(\angle B'BC) = \mu(\angle A'B'B)$

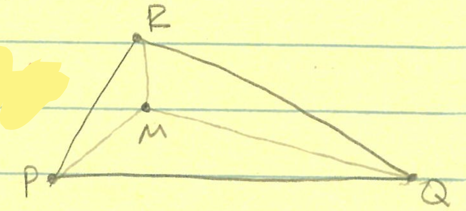
$\Rightarrow \angle B'BC$ and $\angle A'B'B$ are alternating congruent interior angles

$\therefore \; l \parallel l'$ by the Alternate Interior Angles Theorem

∎

Exam ~~3~~ 2? Take Home Problems

1. Prove: If M is any point in the interior of $\triangle PQR$, then $PQ + QR + RP < 2(MP + MQ + MR)$



Proof: Let M be a point in the interior of $\triangle PQR$.

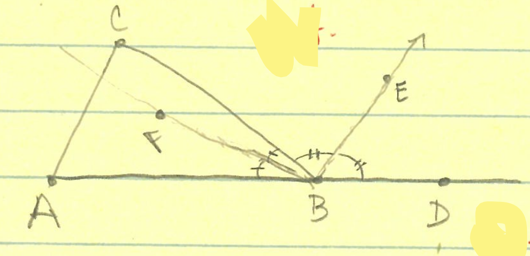
By the Triangle Inequality,

$PQ < MP + MQ$, $QR < MQ + MR$, and $RP < MR + MP$

$\Rightarrow PQ + QR + RP < MP + MQ + MQ + MR + MR + MP$

$\Rightarrow PQ + QR + RP < 2(MP + MQ + MR)$ ∎

2. Given $\triangle ABC$, let D be a point s.t. $A * B * D$. Prove that the angle bisector of the exterior angle at B and the angle bisector of the interior angle at B are perpendicular.



Proof. Let $\triangle ABC$ & pt D be defined as above.

Let $\overrightarrow{BE}$ be the angle bisector of $\angle CBD$

Let $\overrightarrow{BF}$ be the angle bisector of $\angle CBA$. [Show $\mu(\angle FBC) + \mu(\angle CBE) = 90°$]

$\mu(\angle ABC) + \mu(\angle CBD) = 180°$ since they form a linear pair.

$\Rightarrow \mu(\angle ABF) + \mu(\angle FBC) + \mu(\angle CBE) + \mu(\angle EBD) = 180°$ (Angle Add.)

$\Rightarrow 2\mu(\angle FBC) + 2\mu(\angle CBE) = 180°$ by def of angle bisect

$\Rightarrow \mu(\angle FBC) + \mu(\angle CBE) = 90°$

$\Rightarrow \overleftrightarrow{BF} \perp \overleftrightarrow{BE}$

i.e. The angle bisectors are perpendicular ∎