Name: Key

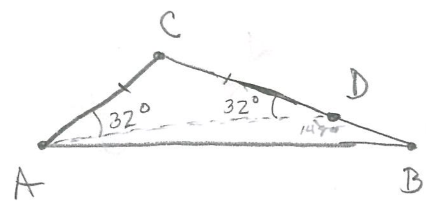
Math 331 Foundations of Geometry – Crawford

Exam 2

19 November 2015

Books and notes (in any form) are not allowed. You may use the provided list of Axioms, Postulates, and Theorems. ***Show all your work.*** Partial credit may be given for written work. GOOD LUCK!

1. (18 pts) Given $\triangle ABC$, let D be a point such that $B * D * C$ and $AC = DC$. Also let $\mu(\angle CAD) = 32°$.



(a). Fill in the blank or explain why there is not enough information to determine: $\mu(\angle ADB) =$ 148°.

(b). Fill in the blank or explain why there is not enough information to determine: $\mu(\angle ACD) =$ ______.

$\sigma(ADC) \leq 180°$ so $\mu(\angle ACD) \leq 116°$

(c). TRUE or FALSE or NOT ENOUGH INFORMATION TO DETERMINE: $CB > AC$.

(d). TRUE or FALSE or NOT ENOUGH INFORMATION TO DETERMINE: (Might not be in Hyperbolic) $AB > AC$.

$\mu(\angle ADB) = 148°$ $\Rightarrow$ $\mu(\angle DBA) + \mu(\angle BAD) \leq 32°$

$\mu(\angle CBA) + \mu(\angle BAD) \leq 32°$

$\mu(\angle CBA) \leq 32° - \mu(\angle BAD)$

$\Rightarrow \mu(\angle ADB) > \mu(\angle CBA)$ $\Rightarrow \mu(\angle CBA) < 32°$

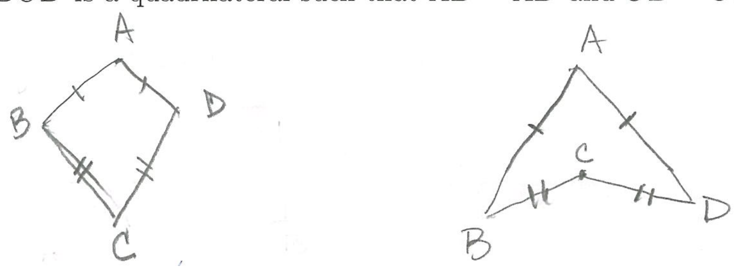
2. (18 pts) Determine whether the following statements are TRUE or FALSE. If the statement is false, sketch a counterexample.

(a). If $\triangle ABC$ and $\triangle DEF$ are triangles such that $\angle BAC \cong \angle EDF, \overline{AC} \cong \overline{DF}$, and $\overline{BC} \cong \overline{EF}$, then $\triangle ABC \cong \triangle DEF$.

A S S

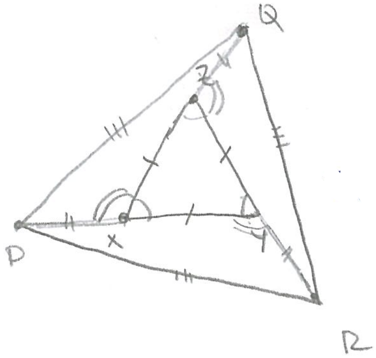
False

(b). If $\square ABCD$ is a quadrilateral such that $AB = AD$ and $CB = CD$, then $\square ABCD$ is a convex quadrilateral.



False

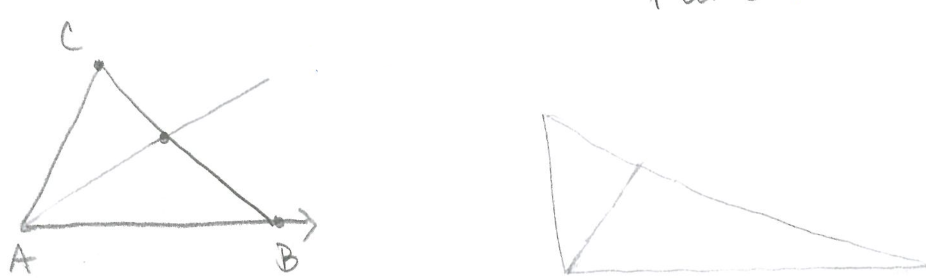
(c). Let $\triangle XYZ$ be an equilateral triangle. Let P, Q, and R be points such that $Y * X * P$, $X * Z * Q$, and $Z * Y * R$ **and** $XP = ZQ = YR$. Then $\triangle PQR$ is equilateral.



True

(d). The angle bisector of $\angle BAC$ intersects $\overline{BC}$ at its midpoint.

False



3. (6 pts) Determine whether the following statements are TRUE or FALSE. [No sketch or explanation necessary.]

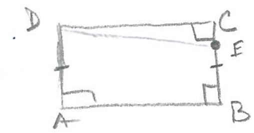
(a). In a model for Neutral Geometry: Given a line l_0 and an external point P_0, if there exists a line m through P_0 such that $m \parallel l_0$, then the Euclidean Parallel Postulate must hold. False

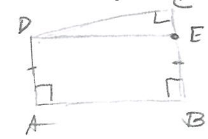
(b). If $\mu(\angle ACB) = \mu(\angle ABC)$, then $AB = AC$. True

(c). If $AB + BC = AC$, then A, B, and C are collinear. True

4. (12 pts) Fill in the blanks to prove Property 4 of Lambert Quadrilaterals (Theorem 4.8.11, part 4).

<u>PROOF</u>: Let $\square ABCD$ be a Lambert quadrilateral with right angles at vertices A, B, and C. [Show $BC \leq AD$.]





BWOC, suppose $\underline{BC > AD}$

Then by point construction, construct a point E between B and C such that $BE = \underline{AD}$.

Then $\square ABED$ is a $\underline{\text{Saccheri}}$ quadrilateral by definition.

Then $\mu(\angle BED) \leq 90°$ by Property 6 of Saccheri Quadrilaterals.

But $\angle BED$ is an exterior angle to $\triangle ECD$.

Therefore $\mu(\angle BED) > \underline{\mu(\angle ECD) = 90°}$ by the Exterior Angle Theorem.

This is a contradiction ($\rightarrow\leftarrow$) to $\underline{\mu(\angle BED) \leq 90°}$

Therefore, $\underline{BC \leq AD}$.

5. (24 pts) <u>Prove 2</u> of the following. Clearly state theorems and properties that you use.
<u>BONUS</u>: You may do (or attempt) all three options and each will be graded out of 12 points. Whichever two you score higher on will be your base grade. Any points from the third problem will be cut in third and added to your base grade.

(a). (Old) **Use the Hinge Theorem** to prove THEOREM 4.2.7 (SSS): If $\triangle ABC$ and $\triangle DEF$ are two triangles such that $\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}$ and $\overline{CA} \cong \overline{FD}$, then $\triangle ABC \cong \triangle DEF$.

(b). (Old) Prove LEMMA 4.5.3 (Two-Angles Sum): If $\triangle ABC$ is any triangle, then $\mu(\angle CAB) + \mu(\angle ABC) < 180°$.

(c). (New) Prove the following corollary to the Alternate Interior Angles Theorem: If l and l' are lines cut by a transversal t in such a way that two nonalternating interior angles on the same side of t are supplements, then l is parallel to l'.

6. (24 pts) The take-home portion of the exam is due Saturday, November 21 by 10am. If you do not turn it in to me in person by Friday, November 21 at 3pm, you may scan it and email it to me. It must be a pdf file, not jpg, and make sure it is legible before sending. The public computer lab in Daniels Hall has scanners that you can use.