

Name: \_\_\_\_\_

Math 331 Foundations of Geometry – Crawford

Exam 1

15 October 2015

Score

1	/12
2	/12
3	/12
4	/6
5	/12
6	/24
7	/24
Total	/100

Books and notes (in any form) are not allowed. You may use the provided list of Axioms, Postulates, and Theorems. **Show all your work.** Partial credit may be given for written work. GOOD LUCK!

1. (12 pts). Let  $A, B, C, D$ , and  $E$  be points such that  $A * B * C$  and the points  $D$  and  $E$  lie on opposite sides of  $\overleftrightarrow{AB}$ . Circle each statement below that **must always** be true.

[Just circle them on this sheet.]

$$D * B * E$$

$$D \notin \overleftrightarrow{AB}$$

$$\overrightarrow{AD} \cap \overrightarrow{CE} = \emptyset$$

$$\overrightarrow{DA} \cap \overrightarrow{CE} = \emptyset$$

$A$  and  $C$  are on opposite sides of  $\overleftrightarrow{DB}$

$A$  and  $C$  are on opposite sides of  $\overleftrightarrow{DE}$

$$AB < AC$$

$$\overline{DE} \cap \overleftrightarrow{AB} \neq \emptyset.$$

2. (12 pts). Given the following model, Points:  $\{A, B, C, D, E, F\}$  Lines:  $\{A, B\}, \{B, C\}, \{D, E, F\}$

- (a). Sketch a schematic diagram of this model.
- (b). Determine which of the Incidence Axioms hold and which of the Parallel Postulates hold.

3. (12 pts).

- (a). Write the contrapositive of the statement: If  $l$  and  $m$  are nonparallel and distinct lines, then  $l \cap m$  contains only one point.
- (b). Write the negation of the statement: If two angles form a linear pair, then the sum of their angle measures is 180.
- (c). Write the negation of the statement: For all Monday holidays, there is no school.

4. (6 pts). Construct a truth table for the statement  $P$  iff  $Q$  (i.e.  $P \Leftrightarrow Q$ ).

5. (12 pts). Given the points  $A(0,0)$  and  $B(3,0)$  in the Cartesian plane,

- (a). Find a point  $C$ , not on the  $y$ -axis, such that  $\triangle ABC$  is isosceles in the taxicab metric. Then find the length using the taxicab metric of the third side.
- (b). Is this isosceles triangle drawn in part (a) unique in the taxicab metric? If not, describe (or sketch) the set of all points  $C$  that will create such an isosceles triangle in the taxicab metric. [You may allow the  $y$ -axis in your consideration of part (b).]

6. (24 pts). Prove 2 of the following. Clearly state theorems and properties that you use.

BONUS: You may do (or attempt) all three options and each will be graded out of 12 points. Whichever two you score higher on will be your base grade. Any points from the third problem will be cut in third and added to your base grade.

- (a). (Old) If  $l$  is any line, then there exists lines  $m$  and  $n$  such that  $l, m,$  and  $n$  are distinct and both  $m$  and  $n$  intersect  $l$ . [This is Theorem 2.6.5 on the list. Prove it using any theorems, postulates, and/or axioms that come before it.]
- (b). (New) Let  $A$  and  $B$  be two distinct points. Prove the following equality of sets.  $\overrightarrow{AB} \cap \overrightarrow{BA} = \overline{AB}$
- (c). (Newish) If  $D$  and  $E$  are two distinct points, then there exists a unique perpendicular bisector for  $\overline{DE}$ . [This is Theorem 3.5.11 on the list. Prove it without using Theorem 3.5.9, but you can use any other theorems, postulates, and/or axioms that come before it.]

7. (24 pts). The take-home portion of the exam is due Sunday, October 18 by noon. If you do not turn it in to me in person by Friday, October 16 at 3pm, you may scan it and email it to me. It must be a pdf file, not jpg, and make sure it is legible before sending. The public computer lab in Daniels Hall has scanners that you can use.