1. A zoo has an open area that contains both zebras and gazelles. The animals are fed two types of food mixes, type A and B. Each day the area is supplied with 80 lbs of Food A and 160 lbs of Food B. Zebras require 2 lbs of Food A and 8 lbs of Food B each day . Gazelles need 5 lbs of Food A and 4 lbs of Food B each day. Determine the maximum number of zebras and gazelles that the zoo can support.

$$\text{Maximize} \quad x+y \qquad \text{Subject to} \quad \begin{array}{rcl} 2x+5y & \leq & 80 \\ 8x+4y & \leq & 160 \\ x, y & \geq & 0 \end{array}$$

Follow the steps below to re-solve the problem algebraically.

(a). If necessary, rewrite the n problem constraints to $\leq$ constraints.

(b). Introduce slack variables for the n problem constraints.

[Note: Since you started with $m =$ _____ decision variables and introduced $n =$ _____ slack variables, you now have $m + n =$ _____ total variables.]

(c). Determine how many possible intersections must be enumerated by determining how many ways to choose m variables from the total $m + n$ to set equal to zero.

Number of possible intersections: $\left(\begin{array}{c} m+n \\ m \end{array} \right) = \dfrac{(m+n)!}{m!\, n!} =$

(d). Enumerate (i.e. solve for) each intersection and determine if it is feasible.

(e). Evaluate the objective at each of the feasible intersections (or extreme points). [Table summary.]

(f). State your optimal solution and interpret it in terms of the original problem scenario.

2. Same as the previous problem, except that the area also contains kangaroos who need $\frac{1}{4}$ lbs of Food A and 5 lbs of Food B.

Clearly state the Linear Program problem and solve it algebraically.

3. Same as the previous problem with zebras, gazelles, and kangaroos, except that the area is required to have at least 3 zebras.

(a). Clearly state the Linear Program problem.

(b). Rewrite the constraints using slack variables. [Don't forget that constraints must be written with $\leq$ before introducing slack variables.]

(c). Determine how many possible intersections need to be checked for feasibility.

[That's a lot to check by hand... let's not do it. Next time we'll learn how to use Excel to solve.]

4. A company owns two factories which produce 3 different kitchen appliances: mixers, toasters, and food processors. Factory 1 produces 800 toasters, 100 mixers, and 200 food processors per day. Factory 2 produces 200 toasters, 100 mixers, and 700 food processors per day. The daily operating costs for these production lines are \$8000 for factory 1 and \$15000 for factory 2. The company has received an order for 1600 toasters, 500 mixers, and 2000 food processors. Determine the number of days each factory should operate to fill the orders at a minimum cost.

$$\begin{array}{lllrcl} \text{Minimize} & 8000x_1 + 15000x_2 & \text{Subject to} & 800x_1 + 200x_2 & \geq & 1600 \\ & & & 100x_1 + 100x_2 & \geq & 500 \\ & & & 200x_1 + 700x_2 & \geq & 2000 \\ & & & x_1, x_2 & \geq & 0 \end{array}$$

Solve the problem algebraically.

Homework: Finish this worksheet and resolve problems #1, 3, 5 from Section 7.2 using the Algebraic Method.