1. A company owns two factories which produce 3 different kitchen appliances: mixers, toasters, and food processors. Factory 1 produces 800 toasters, 100 mixers, and 200 food processors per day. Factory 2 produces 200 toasters, 100 mixers, and 700 food processors per day. The daily operating costs for these production lines are \$8000 for factory 1 and \$15000 for factory 2. The company has received an order for 1600 toasters, 500 mixers, and 2000 food processors. Follow the steps below to set up a mathematical model (linear program) to determine the number of days each factory should operate to fill the orders at a minimum cost.

(a). Write down the decision variables in words: [Hint: What quantities are you trying to determine?]

Let $x =$
$y =$

(b). Fill in the following table to summarize the production and order information.

	x ( )	y( )	Ordered
Toasters			
Mixers			
Food Processors			
Daily Costs			

(c). If Factory 1 operates for x days, what are the operating costs? If Factory 2 operates for y days, what are the operating costs?

What is the total operating cost? Do you want to minimize or maximize this cost?

(d). If Factory 1 operates for x days and Factory 2 operates for y days,

How many total toasters can they produce? Does this total need to be $\geq$ or $\leq$ the number ordered?

How many total mixers can they produce? Does this total need to be $\geq$ or $\leq$ the number ordered?

How many total food processors can they produce? Does this total need to be $\geq$ or $\leq$ the number ordered?

(e). Summarize your work as a linear program. Include any additional reality (or domain) constraints.

Minimize ____________________ Subject to

(f). Using all of the constraints, graph the inequalities and shade the solution region. [Use the given separate graph and try to draw the lines as accurately as possible.]
[Note: The solution region is called the ***feasible region*** because it gives the only possible values of x and y that will satisfy all the constraints.]

(g). Graph the line where the objective function equals $C = 72,000$ [i.e. Graph the line____________________ = 72000.]

Does it intersect the feasible region or lie outside of it?

How many possible values of x and y would be possible solutions for this value of C?

2. A zoo has an open area that contains both zebras and gazelles. The animals are fed two types of food mixes, type A and B. Each day the area is supplied with 80 lbs of Food A and 160 lbs of Food B. Zebras require 2 lbs of Food A and 8 lbs of Food B each day . Gazelles need 5 lbs of Food A and 4 lbs of Food B each day. Follow the steps below to set up a mathematical model (linear program) to determine the maximum number of zebras and gazelles that the zoo can support.

(a). Write down the decision variables in words: [Hint: What quantities are you trying to determine?]

Let x =
y =

(b). Fill in the following table to express the amount of food consumed.

	x ( )	y( )	Delivered
Food A			
Food B			

(c). If there are x zebras and y gazelles, what is the total number of animals in this area?

Do you want to minimize or maximize this total?

(d). If there are x zebras and y gazelles,

How much of Food A do they consume? Does this total need to be $\geq$ or $\leq$ the amount delivered?

How much of Food B do they consume? Does this total need to be $\geq$ or $\leq$ the amount delivered?

(e). Summarize your work as a linear program. Include any additional reality (or domain) constraints.

Maximize ___________ Subject to

(f). Using all of the constraints, graph the inequalities and shade the feasible region on the separate sheet of paper.

3. [*Continuation of Problem 2*] Since we want to maximize the objective $x + y$, let's set it equal to several values (C) and see if we can graphically determine the solution where it is largest.

For each value of C given in the table below:

(a). Fill in the second and third columns of the table.

Value of the Objective C	$x + y = C$	(equivalently) $y = -x + C$	Number of Possible Solutions (x, y)
10	$x + y = 10$	$y = -x + 10$	
20			
25			
30			

(b). Graph the line $x + y = C$ (equivalently $y = -x + C$) on your graph and label it with the value of C.
[Try to do this as accurately as possible.]

(c). Observe whether the line intersects the feasible region or lies outside of the feasible region.
Fill in the fourth column in the table by stating how many values of x and y would then be possible solutions for that value of C. [You can state *zero*, *one*,... or *infinitely many*]

4. [*Continuation*] Based on the table above, what is the largest value of C that has possible solutions for (x, y) in the feasible region? [This is your maximal solution for the objective.]

(a). What are the values of x and y for this value of C? [Hint: Which two lines intersect?]

(b). If you let C be one integer less than the answer above, explain why that will not give the maximal solution.

(c). If you let C be one integer greater than the answer above, explain why that will not give the maximal solution.

5. [*Continuation*] For the value of C found in #4, determine the solution to this linear programming problem.
i.e. Find the maximum number of zebras (x) and gazelles (y) that the zoo can support.
[Hint: Look at your answer in #4(a).]

6. Given the following linear program, repeat the steps of Problems#2-5 above to graphically determine the maximum and minimum of the objective function.

Optimize $2x + y$ Subject to

$$\begin{aligned} x + 3y &\leq 6 \\ x + y &\leq 4 \\ x &\geq 1 \\ y &\geq 0 \end{aligned}$$

Value of the Objective C	$2x + y = C$	(equivalently) $y = -2x + C$	Number of Possible Solutions (x, y)
0			
1			
2			
3			
4			
5			
6			
7			
8			
9			

Which value of C is the largest possible and still gives values of x and y in the feasible region? What are the values of x and y for this value of C?

Which value of C is the smallest possible and still gives values of x and y in the feasible region? What are the values of x and y for this value of C?

7. Based on the work above, suggest a reasonable conclusion about the **<u>location</u>** of the solutions that optimize the objective function.