1. Preparatory Questions

(a). (Calculus I) Find the critical numbers of $f(x) = 3x^3 - 4x$ and determine any the ***location*** of any maximum or minimum values of f.

(b). (Calculus III) In order to find the critical points of $f(x, y) = x^2 + xy + y^2 + y$, what two conditions must be satisfied simultaneously? Then find the critical points.

(c). How are critical numbers or points related to possible maximum and minimum values of a function?

(d). Graph $f(x) = |x|$.

Find $f'(x)$ and graph it. [Hint: Write $f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$]

Is f continous? Is $f'(x)$ continuous?

2. Given the data from the example in class:

x	1	10	25
y	4	20	48

Use Criterion: <u>MINIMIZE THE SUM OF THE ABSOLUTE DEVIATIONS</u> to fit a line $f(x) = ax + b$ to the data.
[Complete the steps below to see how the process is done.]

(a). Let $S = \sum_{i=1}^{3} |y_i - f(x_i)|$ [i.e. sum of the absolute deviations]

Write down the explicit form of S found in class for this data set. [Expression (*) from the notes.]

(b). S is a function of which two variables?

(c). In order to minimize S, what derivative(s) do we need to take? Why is this problematic?
[Don't try to find the derivative(s)!]

So, while this criterion makes a lot of sense, it becomes mathematically very complicated. Let's try something else ⇒

3. Given the data from the example in class:

x	1	10	25
y	4	20	48

Use the Least-Squares Criterion: <u>Minimize the Sum of the Deviations Squared</u> to fit a line $f(x) = ax + b$ to the data.

[Complete the steps below to see how the process is done.]

(a). Let $S = \sum_{i=1}^{3} |y_i - f(x_i)|^2 = \sum_{i=1}^{3}(y_i - f(x_i))^2 = \sum_{i=1}^{3}(y_i - ax_i - b)^2$ [i.e. sum of the deviations squared]

Why is it okay to replace the absolute values with parentheses?

Write out S explicitly for the data set given above. i.e., Plug in the actual data points. [Do not expand/foil]

(b). S is a function of which two variables?

(c). In order to minimize S, what derivative(s) do we need to take? Why will this step be easier than for the previous problem?

(d). Compute $\dfrac{\partial S}{\partial a}$ and $\dfrac{\partial S}{\partial b}$. [Don't forget the chain rule.]

(e). Set $\dfrac{\partial S}{\partial a} = 0$ and $\dfrac{\partial S}{\partial b} = 0$ and simplify the equations. [Note: You can divide both side by -2 first.]

How many equations and how many unknowns do you have?

(f). Solve the equations in step (e) for the parameters a and b.

(g). Write down the best fit line (using the Least-Squares Criterion) in the form $f(x) = ax + b$.