- If a population changes proportional to the population size itself, then the model is given by $\Delta p_n =$ $\underline{\quad kp_n \quad}$

  If $k > 0$, then the population will $\underline{\quad \text{grow} \quad}$. If $k < 0$, then the population will $\underline{\quad \text{decrease} \quad}$.

- The interaction between a population with itself is represent by a $\underline{\quad \text{product} \quad}$, such as p_n^2 or $p_n(M - p_n)$.

  The logistic model, $\Delta p_n = kp_n(M - p_n), k > 0$ can be expanded as $\Delta p_n = kMp_n -$ $\underline{\quad kp_n^2 \quad}$.

  The negative in front of the interaction p_n^2 term indicates that the interaction of the population with itself will cause the population size to $\underline{\quad \text{decrease} \quad}$.

  [i.e. The interaction has a detrimental effect on the population size since they are all competing for the same resources.]

For the following two problems, use **positive** coefficients k_1, k_2, k_3, and k_4 as needed.

1. Competing Species: Two species which compete for the same resources, lives in the same habitat, etc. (e.g. foxes and coyotes or owls and hawks)

Let f_n and c_n be the population sizes of foxes and coyotes, respectively, at time interval n.

Complete the following steps to construct a model for this system of two populations

(a). Assume that each population would grow unconstrained (proportional to the population size) in the absence of the other species. Write down two equations for Δf_n and Δc_n that would model this assumption (use the coefficients k_1 and k_2).

$$\Delta f_n = \underline{\quad k_1 f_n \quad}$$

$$\Delta c_n = \underline{\quad k_2 c_n \quad}$$

(b). In mathematical notation, what type of term represents the interaction between the two populations (foxes and coyotes)?

If we now assume that the effect on the population change is proportional to the number of possible interactions between foxes and coyotes, what would this term look like?

Would you add or subtract a term like this to the model you started above? Why? [Hint: Will the interaction between foxes and coyotes be advantageous or detrimental to each population? Go back and read the description of "competing species," if needed.]

Would you use the same proportionality constant? Why or why not?

Now modify the model you started in part (a) to take into account these interactions (use the coefficients k_3 and/or k_4 and appropriate signs).

$$\Delta f_n = \underline{\quad k_1 f_n - k_3 f_n c_n \quad}$$

$$\Delta c_n = \underline{\quad k_2 c_n - k_4 f_n c_n \quad}$$

2. Predator-Prey Species: Two species where one species is the primary food source (prey) of the other species (predator) and they live in the same habitat (e.g. coyotes and rabbits, owls and mice).

Assumptions:

- In the absence of the predator, the prey population would grow unconstrained

  because ___they are not being hunted/eaten___.

- In the absence of the prey, the predator population will decrease

  because ___they do not have their primary food source___.

- The interaction between the predator and the prey will cause the prey population to ___decrease___.

- The interaction between the predator and prey will cause the predator population to ___increase___.

Write down two equations for the coyotes Δc_n and rabbits Δr_n that model this situation. Be sure to include coefficients and appropriate signs. Should any of the coefficients (proportionality constants) be the same? Why or why not?

Write the systems found in problems 1 and 2 in their alternate forms $f_{n+1} = \ldots$.

<u>COMPETING SPECIES MODEL</u>

<u>PREDATOR-PREY MODEL</u>