Recall, the simplest model assumed that the change in the amount to be proportional to the amount present, i.e. $\Delta a_n = ka_n$

which led to the model $a_{n+1} = a_n + ka_n = (1+k)a_n$.

Let $r = 1 + k$ to obtain the general dynamical system: $\boxed{\begin{array}{rcl} a_{n+1} & = & ra_n \\ a_0 & & \text{given} \end{array}}$

1. **<u>r > 1</u>** (e.g. *Unconstrained* Aphid Population Example): $\begin{array}{rcl} a_{n+1} & = & 1.6198a_n \\ a_0 & = & 34,285 \end{array}$

Iterate the following system and sketch the graph. What happens to the size of the aphid population as time goes on? [i.e. What is the limit as $n \to \infty$?]

2. **<u>r = 1</u>** Iterate the following system and sketch the graph. Describe what happens as $n \to \infty$. $\begin{array}{rcl} a_{n+1} & = & a_n \\ a_0 & = & 5 \end{array}$

3. **<u>0 < r < 1</u>** (e.g. Ibuprofen Example): $\begin{array}{rcl} a_{n+1} & = & 0.8a_n \\ a_0 & = & 400 \end{array}$

Iterate the following system and sketch the graph. What happens to the amount of ibuprofen in one's system as time goes on? [i.e. What is the limit as $n \to \infty$?]

4. **<u>r = 0</u>** Iterate the following system and sketch the graph. Describe what happens as $n \to \infty$. $\begin{array}{rcl} a_{n+1} & = & 0 \\ a_0 & = & 5 \end{array}$

5. What if **r is negative**? (rhetorical)
Iterate the following systems up to $n = 5$ and sketch a graph of the iterations. Then describe the behavior.

(a). $\mathbf{-1 < r < 0}$ $\quad a_{n+1} = -\frac{2}{3}a_n$, $\quad a_0 = 18$

(b). $\mathbf{r = -1}$ $\quad a_{n+1} = -a_n$, $\quad a_0 = 3$

(c). $\mathbf{r < -1}$ $\quad a_{n+1} = -2a_n$, $\quad a_0 = 1$

6. Summarize the results for the linear dynamical system of the form: $a_{n+1} = ra_n$ (with a_0 given)

r	Behavior (e.g. constant, growth, decay, oscillation)	Limiting Value or DNE
$r > 1$		
$r = 1$		
$0 < r < 1$		
$r = 0$		
$-1 < r < 0$		
$r = -1$		
$r < -1$		

Note:

If $|r| < 1$ solutions decay to 0 .

If $|r| > 1$ solutions are unbounded .