1. LOST: A plane crashes on a deserted, isolated island and there are 120 survivors. Unfortunately, one of the survivors is already infected with a virus (not deadly, if treated in a timely manner). Assume that the virus spreads so that the ***change in the number of infected people is proportional to number of interactions between the infected and non-infected (susceptible) people.***

Let I_n be the number of people infected.

(a). What is the expression for the number of people not infected?

(b). What is the expression representing the interaction between infected and non-infected persons?

(c). Using the assumption stated above, write down the difference equation that models the change in the number of infected people. [Use k for the proportionality constant.]

(d). What is the initial condition I_0?

(e). After 1 day, 2 people are infected. Determine the proportionality constant. [Note: The proportionality constant gives the proportion of interactions that result in the virus spreading (i.e. the rate at which the virus spreads).]

(f). If you haven't done so already, write your model in the form $I_{n+1} = \dots$ with the appropriate initial condition.

(g). Iterate the model to determine how many days they have to be rescued before everyone becomes infected.

2. Simplest Model – Unconstrained Growth

Let p_n represent a single population at time interval n and write down the model that represents the assumption that the population ***grows*** proportional to the size of the population itself.

$$\Delta p_n = \underline{\hspace{2cm}}, k > 0$$

(a). We assumed $k > 0$. Can you think of a situation in which this assumption would lead to $k < 0$ for a population?

(b). The population p_n must always be positive. And if $k > 0$, the change in the population $\Delta p_n = kp_n$ is always ______________ , representing growth.

(c). This model is good for a while, but eventually as time goes on this model will become invalid since the population cannot grow ______________ .

3. Better Model – Constrained Growth – Carrying Capacity

$$\Delta p_n = kp_n(M - p_n), \quad k > 0$$

(a). The population p_n must always be ______________ . [sign?]

(b). The product $p_n(M-p_n)$ represents the ________________ of the members of the population with themselves (e.g. competition).

(c). If the population size is smaller than the carrying capacity M, then $M - p_n$ is positive. Hence, the product $kp_n(M - p_n)$ is also ______________ , so that the change Δp_n is ______________ , representing population growth.

This makes sense because if the population is less than the carrying capacity M, the population has enough resources to continue ______________ .

(d). If the population size is larger than the carrying capacity M, then $M - p_n$ is ______________ . Hence, the product $kp_n(M - p_n)$ is also ______________ , so that the change Δp_n is ______________ , representing population decline.

This makes sense because if the population is greater than the carrying capacity M, the population does not have enough resources and must ______________ .

4. Possibly Better Model – Carrying Capacity and Extinction

Let's try to modify the previous model to account for the idea of extinction: ***If the population size of a species falls below a minimum value m, then it will die off and become extinct.***

(a). Write down a term involving p_n and m that captures both of the statements given below.

(1) If the population size is less than the minimum m, we need a term that is negative so that it will cause the population to decrease.
(2) If the population size is greater than the minimum m, we need that *same* term to be positive so that it will cause the population to increase.

(b). From the logistic model in #3, the term $M - p_n$ represents how much the population size differs from the ____________________ M.

(c). Then your answer in (a) ____________ represents how much the population size differs from the extinction level ________ .

(d). If the population changes proportional to the interaction between these two sub-populations in (b) and (c), write down a model for this situation.

$$\Delta p_n = k\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \quad k > 0$$

Write the model in the standard form for iteration

$$p_{n+1} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \quad k > 0$$

$$p_0 \text{ given}$$

HOMEWORK:

- (Excel) Consider a population of whales where the carrying capacity is 5000, the extinction level is 500, and $k = 0.0001$. Iterate the model from #4 up to $n = 40$ for the following initial conditions $a_0 = 499, 500, 501, 1000, 5000, 6000$. Graph the solutions all on the same graph and include a legend. [Note: If any of the populations become negative, replace those values with 0 before graphing.]
- Expand the terms (e.g. distribute, FOIL, etc.) in both the Constrained Growth (#3) and the Carrying Capacity-Extinction (#4) Models. [Use the p_{n+1} form.] Compare the two models in this form and determine whether you think these two models are significantly different or essentially the same. Justify your answer. [Hint: You may want to consider the graphs of both models if you let $y = p_{n+1}$ and $x = p_n$.]
- Section 1.2, p. 16: #3-4, 6-8
  Section 1.3, p. 34: #13, 14[Note: You may get "interesting" behavior on 14. We'll discuss that later.]