Name: ______________________________ **Exam 2**

Math 311 Mathematical Modeling – Crawford **11 May 2017**

Books and notes (in any form) are not allowed. You may use a calculator. ***Show all your work*** and clearly indicate your answers – partial credit may be given for written work. Good Luck!

Part A: Computers are not allowed on Part A. You must turn in Part A before beginning on Part B.

1. (12 pts) Suppose you want to fit a curve of the form $f(x) = A\sqrt{x}$ to a collection of m data points. Apply the Least-Squares Criterion to ***derive*** a formula for the parameter A. [You must show all work in deriving the formula.]

2. (18 pts) A company produces two types of drones, which are assembled on 2 different assembly lines. Line 1 can assemble 30 units of the Basic Model and 40 units of the Advanced Model per hour. Line 2 can assemble 150 units of the Basic Model and 40 units of the Advanced Model per hour. The company needs to produce at least 270 units of the Basic Model and 200 units of the Advanced Model to fill an order. The cost to run Line 1 is $200 per hour and Line 2 is $300 per hour.

(a). Formulate a linear program to determine how many hours they should operate each assembly line should operate to minimize cost. [Clearly indicate the Decision Variables, Objective, and Constraints.]

(b). Sketch (and shade) the feasible region.

(c). Solve the linear programming problem graphically. [Write your final answer in the context of the problem.]

3. (12 pts) Given the following linear program

Maximize $2x + 3y$ subject to

$$\begin{aligned} 2x + 3y &\geq 6 \\ 3x &\leq 15 \\ -x + y &\leq 4 \\ 2x + 5y &\leq 27 \\ x, y &\geq 0 \end{aligned}$$

(a). Begin the process of solving algebraically by introducing appropriate slack variables.

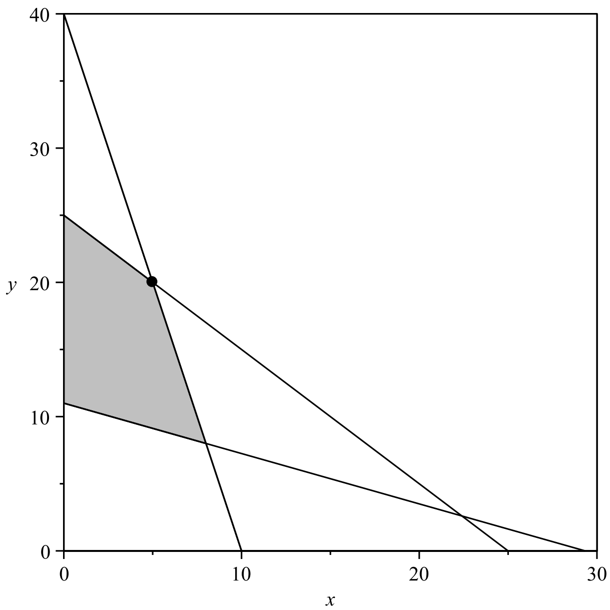
(b). How many possible intersection points are there?

(c). *Using the equations* introduced in part **(a)**, find **one** intersection point (other than the origin (0,0)) and determine whether it is feasible or not feasible. Clearly indicate why you have concluded that it is feasible or not.

4. (18 pts) The feasible region for the following linear program is given in the graph below.

Maximize $4x + 2y$ subject to

$$\begin{aligned} x + y &\leq 25 \\ 4x + y &\leq 40 \\ 3x + 8y &\geq 88 \\ x, y &\geq 0 \end{aligned}$$



The point $(5, 20)$ maximizes the objective (with a value of 60).

(a). If the objective function is changed to $c_1 x + 2y$, perform a sensitivity analysis of the optimal solution to determine the range of allowable c_1 values.

(b). If the second constraint is changed to $4x + y \leq b_2$, perform a sensitivity of the optimal solution to determine the range of allowable b_2 values. Determine the associated shadow price(s).

5. (10 pts) Under certain conditions the heat loss of a warm-blooded animal is proportional to the exposed body surface area.

(a). Use this assumption and geometric similarity to relate the heat loss of an 18 ft tall giraffe to a 12 ft tall giraffe.

(b). Assuming the amount of energy needed to maintain a constant body temperature is proportional to surface area, develop a proportionality model based on geometric similarity that relates the amount of energy needed to the weight of the animal. Assume a constant weight density.

<u>Part B</u>: You must turn in Part A before beginning Part B of the exam. Books and notes (in any form) are not allowed. You may use a computer and calculator for Part B. Clearly indicate your answers.

6. (16 pts) It is desired to approximate the least-squares fit of the model $y = Ae^{Bx}$ using a transformation.

(a). Determine the transformed equation that you would use.

(b). Using your transformation, ***modify*** the formulas for the slope a and intercept b (given below) found by fitting a straight line via the least-squares criterion, to find the parameters of your transformed model.

$$a = \frac{m\sum x_i y_i - \sum x_i \sum y_i}{m\sum x_i^2 - \left(\sum x_i\right)^2} \qquad b = \frac{\sum x_i^2 \sum y_i - \sum x_i y_i \sum x_i}{m\sum x_i^2 - \left(\sum x_i\right)^2}$$

(c). **<u>Use your results of part (b)</u>** and Excel to fit the model $y = Ae^{Bx}$ to the following data. Show intermediate sums in Excel. Be sure to write your final model.

x	0.28	0.55	0.68	0.99	1.15	1.39
y	225	166	157	126	96	71

7. (16 pts) Use Excel Solver to solve following linear programming problem. If a solution does not exist, clearly state so.

Minimize $10x + 30y + 35z$ subject to

$$\begin{aligned} x + y + z &\leq 250 \\ x + y + 2z &\geq 150 \\ 2x + y + z &\leq 180 \\ x, y &\geq 0 \end{aligned}$$

[Clearly indicate the solution and the minimal value.]

You may download the template from the class website http://crawford.elmhurst.edu/.

[Save the file as E2_YourLastName.xlsx and email it to crawford@elmhurst.edu]