<u>THEOREM</u> For all integers a and b, if a and b are positive and a divides b, then $a \leq b$.

<u>PROOF</u> Let a and b be __positive integers__ and let a divide b. [Show __$a \leq b$__.]

By __definition of divisibility__, there exists an integer k such that __$b = ak$__.

Then since b is positive, then ak is __positive__.

Then by Theorem T25 (App. A), since $ak > 0$ and a is positive, then __k is also positive__.

Since k is a positive integer, it follows that $1 \leq k$.

Multiplying both sides by a, which we know is positive, gives

__$a \leq ak$__ by T20 (App. A).

Therefore, __$a \leq b$__ by substitution. ■

<u>THEOREM</u> The only divisors of 1 are 1 and -1.

<u>PROOF</u> Let m be a ___divisor___ of 1. [Show that m must be ___1 or -1___ .]

Then by definition of divisibility, m is an ___integer___ and there exists an integer k such that $1 =$ ___mk___.

By Theorem T25 (App. A), either m and k are both ___positive___ or they are both ___negative___ .

<u>Case 1</u>: m and k are both positive.

Since m and 1 are positive integers and m divides 1, by the previous theorem, ___$m \leq 1$___ .

The only way for positive integer to be less than or equal to 1, is for the integer to be ___1___ .

Therefore, $m = 1$.

<u>Case 2</u>: m and k are both ___negative___ .

Then by Theorem T12 (App. A), $(-m)(-k) =$ ___mk___ $= 1$.

Thus, by definition, $-m$ is a divisor of ___1___ .

Also, since m is negative, $-m$ is ___positive___ .

Hence $-m$ is a positive divisor of 1, and by the previous theorem, ___$-m$___ ≤ 1.

By the same reasoning as above, $-m = 1$ and therefore, ___$m = -1$___ .

Since these are the only two possibilities, $m = 1$ or $m = -1$, then the only divisors of 1 are ___1 or -1___ . ■

Homework Section 4.4

Read p. 192 Caution! paragraph.

p. 197: #4, 8, 14, 15, 18, 20, 22, 25.